MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

PG and Research Department of Mathematics

II B.Sc. Mathematics– Semester - IV

E-Notes (Study Material)

Elective: Mathematical Statistics II	**Code:23UEMA43**
Unit:III- F-distribution –Applications of F-distribution	
Learning Objectives: Explain the derivation and properties of the F-distribution.	
Course Outcome: Derive the various measures of F distributions	

Overview: F distribution, Application of F distribution

Unit - 3

F - Distribution

* If ψ_1^2 and ψ_2^2 are two independent chi square variate with γ_1 and γ_2 degrees of freedom respectively.

The F-distribution F-statistics is defined by

$$F = \frac{\psi_1^2/\gamma_1}{\psi_2^2/\gamma_2}$$

In other words F is defined as the ratio of two independent Chi-square variate divided by their respective degrees of freedom and it follows snedecors F distribution with γ_1 and γ_2 degrees of freedom.

The probability function is given by.

$$f(F) = \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1/2} F^{\frac{\gamma_1}{2}-1}}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)\left(1+F\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1+\gamma_2}{2}}}, \quad 0 \le F \le \infty$$

Derivatives of Snedecors 'F' distribution.

10m

Let ψ_1^2 and ψ_2^2 are two independent chi-square variate γ_1 and γ_2 degrees of freedom respectively.

The joined probability differential is given by

$$dF(\psi_1^2, \psi_2^2) = dF(\psi_1^2)\, dF(\psi_2^2)$$

$$= \frac{1}{2^{\gamma_1/2}\Gamma\frac{\gamma_1}{2}} e^{\frac{-\psi_1^2}{2}} (\psi_1^2)^{\frac{\gamma_1}{2}-1} d\psi_1^2$$

$$\frac{1}{2^{\gamma_2/2}\Gamma\frac{\gamma_2}{2}} e^{-\frac{\psi_2^2}{2}} (\psi_2^2)^{\frac{\gamma_2}{2}-1} d\psi_2^2$$

$$= \frac{1}{2^{\frac{\gamma_1+\gamma_2}{2}} \Gamma\frac{\gamma_1}{2} \Gamma\frac{\gamma_2}{2}} e^{-\left(\frac{\psi_1^2+\psi_2^2}{2}\right)} (\psi_1^2)^{\frac{\gamma_1}{2}-1} (\psi_2^2)^{\frac{\gamma_2}{2}-1} d\psi_1^2 \, d\psi_2^2$$

Let us make the transformation

$$F = \frac{\psi_1^2/\gamma_1}{\psi_2^2/\gamma_2} \text{ and } u = \psi_2^2 \qquad p(x) = \frac{1}{2^{n/2}\Gamma n/2} e^{-x/2} x^{\frac{n}{2}-1}$$

where

$$dP(x_1) = \frac{1}{2^{n_1/2}\Gamma\frac{n_1}{2}} e^{-\frac{x_1}{2}} x_1^{\frac{n_1}{2}-1} dx_1$$

$$0 \le F < \infty$$

$$0 < u < \infty \qquad dP(x_2) = \frac{1}{2^{\frac{n_2}{2}}\Gamma\frac{n_2}{2}} e^{-\frac{x_2}{2}} x_2^{\frac{x_2}{2}-1} dx_2$$

$$F = \frac{\psi_1^2}{\gamma_1} \times \frac{\gamma_2}{\psi_2^2} = \frac{\psi_1^2}{\gamma_1} \times \frac{\gamma_2}{u}$$

$$\frac{Fu\gamma_1}{\gamma_2} = \psi_1^2$$

Jacobian transformation is given by

$$J = \frac{\partial(\psi_1^2, \psi_2^2)}{\partial(F,u)}$$

$$= \begin{vmatrix} \frac{\partial\psi_1^2}{\partial F} & \frac{\partial\psi_1^2}{\partial u} \\ \frac{\partial\psi_2^2}{\partial F} & \frac{\partial\psi_2^2}{\partial u} \end{vmatrix} = \begin{vmatrix} \frac{u\gamma_1}{\gamma_2} & \frac{F\gamma_1}{\gamma_2} \\ 0 & 1 \end{vmatrix}$$

$$J = \frac{u\gamma_1}{\gamma_2} - 0$$

$$J = \frac{u\gamma_1}{\gamma_2}$$

Thus the distribution of transformed variate is $dG(F,u) = dF(\psi_1^2)\,dF(\psi_2^2)\,|J|$

$$= \frac{1}{2^{\frac{\gamma_1+\gamma_2}{2}} \Gamma\frac{\gamma_1}{2} \Gamma\frac{\gamma_2}{2}} e^{-\frac{1}{2}\left(\frac{Fu\gamma_1}{\gamma_2}+u\right)} \left(\frac{Fu\gamma_1}{\gamma_2}\right)^{\frac{\gamma_2}{2}-1} \frac{\gamma_1}{\gamma_2} u \, dF du$$

$$= \frac{1}{2^{\frac{\gamma_1+\gamma_2}{2}} \Gamma\frac{\gamma_1}{2} \Gamma\frac{\gamma_2}{2}} e^{-\frac{u}{2}\left(F\frac{\gamma_1}{\gamma_2}+1\right)} F^{\frac{\gamma_1}{2}-1} u^{\frac{\gamma_1}{2}-1+\frac{\gamma_2}{2}-1+1} \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1}{2}-1+1} dF\,du$$

$$= \frac{1}{2^{\frac{\gamma_1+\gamma_2}{2}}\Gamma\frac{\gamma_1}{2}\Gamma\frac{\gamma_2}{2}} e^{-\frac{u}{2}\left(F\frac{\gamma_1}{\gamma_2}+1\right)} F^{\frac{\gamma_1}{2}-1} u^{\frac{\gamma_1}{2}+\frac{\gamma_2}{2}-1} \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1}{2}} dF\,du$$

Integrating w.r to 'u' over the range 0 to ∞, $0 < F < \infty$, the distribution of F becomes

$$dG(F,u) = \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1/2} F^{\frac{\gamma_2}{2}-1}}{2^{\frac{\gamma_1+\gamma_2}{2}}\Gamma\frac{\gamma_1}{2}\Gamma\frac{\gamma_2}{2}} dF \int_0^\infty e^{-u/2\left(F\frac{\gamma_1}{\gamma_2}+1\right)} u^{\left(\frac{\gamma_1+\gamma_2}{2}-1\right)} du$$

$$\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$$

$$= \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1/2} F^{\gamma_2/2-1}}{2^{\frac{\gamma_1+\gamma_2}{2}}\Gamma\frac{\gamma_1}{2}\Gamma\frac{\gamma_2}{2}} \frac{\Gamma\frac{\gamma_1+\gamma_2}{2}}{\left[\frac{1}{2}\left(F\frac{\gamma_1}{\gamma_2}+1\right)\right]^{\frac{\gamma_1+\gamma_2}{2}}} dF$$

$$= \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1}{2}} F^{\frac{\gamma_2}{2}-1}}{2^{\frac{\gamma_1+\gamma_2}{2}}\Gamma\frac{\gamma_1}{2}\Gamma\frac{\gamma_2}{2}} \frac{\Gamma\frac{\gamma_1+\gamma_2}{2}}{\left(\frac{1}{2^{\frac{\gamma_1+\gamma_2}{2}}}\right)\left(1+F\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1+\gamma_2}{2}}} dF$$

$$= \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1/2} F^{\gamma_1/2-1}}{\Gamma\frac{\gamma_1}{2}\Gamma\frac{\gamma_2}{2}} \frac{\Gamma\frac{\gamma_1+\gamma_2}{2}}{\left(1+F\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1+\gamma_2}{2}}} dF$$

$$= \frac{\Gamma\frac{\gamma_1+\gamma_2}{2}}{\Gamma\frac{\gamma_1}{2}\Gamma\frac{\gamma_2}{2}} \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1}{2}} F^{\gamma_1/2-1}}{\left(1+F\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1+\gamma_2}{2}}} dF$$

$$dG(F,u) = \frac{1}{\beta\left(\frac{\gamma_1}{2},\frac{\gamma_2}{2}\right)} \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1/2} F^{\gamma_1/2-1}}{\left(1+F\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1+\gamma_2}{2}}}$$

$$\therefore f(F) = \frac{1}{\beta\left(\frac{\gamma_1}{2},\frac{\gamma_2}{2}\right)} \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1/2} F^{\gamma_1/2-1}}{\left(1+F\frac{\gamma_1}{\gamma_2}\right)^{\frac{\gamma_1+\gamma_2}{2}}}$$

Which is the required probability function of 'F' distribution with γ_1 and γ_2 degrees of freedom.

Constant of F distribution

16.33 → Eg 16.20 – 16.24 (5) Derive

$$\mu_r' \text{ (about origin)} = E(F^r) = \int_0^\infty F^r f(F)\, dF$$

$$= \int_0^\infty F^r \frac{1}{\beta\left(\frac{r_1}{2}, \frac{r_2}{2}\right)} \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} F^{\frac{r_1}{2}-1}}{\left(1 + F\frac{r_1}{r_2}\right)^{\frac{r_1+r_2}{2}}} dF$$

Let $y = \frac{r_1}{r_2} F$

$F = y\frac{r_2}{r_1}$

$dF = \frac{r_2}{r_1} dy$

sub

$$= \int_0^\infty \frac{y^r\left(\frac{r_2}{r_1}\right)^r \left(\frac{r_1}{r_2}\right)^{r_1/2} \left(y\frac{r_2}{r_1}\right)^{\frac{r_1}{2}-1}}{\beta\left(\frac{r_1}{2}, \frac{r_2}{2}\right)\left(1 + y\frac{r_2}{r_1} \times \frac{r_1}{r_2}\right)^{\frac{r_1+r_2}{2}}} \left(\frac{r_2}{r_1}\right) dy$$

$$= \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2}}{\beta\left(\frac{r_1}{2}, \frac{r_2}{2}\right)} \int_0^\infty \frac{y^r \left(\frac{r_2}{r_1}\right)^{r + \frac{r_1}{2} - 1} y^{\frac{r_1}{2}-1}}{(1+y)^{\frac{r_1+r_2}{2}}} \left(\frac{r_2}{r_1}\right) dy$$

$$= \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2}}{\beta\left(\frac{r_1}{2}, \frac{r_2}{2}\right)} \int_0^\infty \frac{y^{r + \frac{r_1}{2} - 1} \left(\frac{r}{2}\right)^{\frac{r_1}{2} + r}}{(1+y)^{\frac{r_1+r_2}{2} + r - r}} dy$$

Add and sub (r)

$$= \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} \left(\frac{r_2}{r_1}\right)^{r + \frac{r_1}{2}}}{\beta\left(\frac{r_1}{2}, \frac{r_2}{2}\right)} \int_0^\infty \frac{y^{r + \frac{r_1}{2} - 1}}{(1+y)^{\left(\frac{r_1}{2} + r\right) + \left(\frac{r_2}{2} - r\right)}} dy$$

$$\therefore \int_0^\infty \frac{x^{a-1}}{(1+x)^{a+b}} dx = \beta(a,b)$$

where $a = \frac{r_1}{2} + r$, $b = \frac{r_2}{2} - r$

$$\mu_r' = \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} \left(\frac{r_1}{r_2}\right)^{-r - \frac{r_1}{2}}}{\beta\left(\frac{r_1}{2}, \frac{r_2}{2}\right)} \beta\left(\frac{r_1}{2} + r, \frac{r_2}{2} - r\right)$$

$$= \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{-r}}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)} \beta\left(\frac{\gamma_1}{2}+r, \frac{\gamma_2}{2}-r\right)$$

$$\mu_r' = \frac{\left(\frac{\gamma_2}{\gamma_1}\right)^{r}}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)} \beta\left(\frac{\gamma_1}{2}+r, \frac{\gamma_2}{2}-r\right) \qquad \beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$$

In particular $r = 1$

$$\mu_1' = \frac{\left(\frac{\gamma_2}{\gamma_1}\right)}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)} \beta\left(\frac{\gamma_1}{2}+1, \frac{\gamma_2}{2}-1\right)$$

(mean)

$$= \frac{\left(\frac{\gamma_2}{\gamma_1}\right)}{\frac{\Gamma\frac{\gamma_1}{2}\,\Gamma\frac{\gamma_2}{2}}{\Gamma\left(\frac{\gamma_1}{2}+\frac{\gamma_2}{2}\right)}} \frac{\Gamma\left(\frac{\gamma_1}{2}+1\right)\Gamma\left(\frac{\gamma_2}{2}-1\right)}{\Gamma\left(\frac{\gamma_1}{2}+1+\frac{\gamma_2}{2}-1\right)} \qquad \because \Gamma n+1 = n\Gamma n$$

$$\Gamma n = \Gamma n-1\,(n-1)$$

$$= \frac{\left(\frac{\gamma_2}{\gamma_1}\right)}{\Gamma\frac{\gamma_1}{2}\,\Gamma\frac{\gamma_2}{2}} \frac{\gamma_1}{2}\Gamma\frac{\gamma_1}{2}\,\Gamma\left(\frac{\gamma_2}{2}-1\right)$$

$$= \frac{\left(\frac{\gamma_2}{\gamma_1}\right)}{\left(\frac{\gamma_2}{2}-1\right)\Gamma\left(\frac{\gamma_2}{2}-1\right)} \frac{\gamma_1}{2}\Gamma\left(\frac{\gamma_2}{2}-1\right)$$

$$= \frac{\gamma_2}{\gamma_1} \times \frac{1}{\frac{\gamma_2}{2}-1} \times \frac{\gamma_1}{2}$$

$$\mu_1' = \frac{\gamma_2}{\frac{(\gamma_2-2).2}{2}} = \frac{\gamma_2}{\gamma_2-2}$$

put $r = 2$

$$\mu_2' = \frac{\left(\gamma_2/\gamma_1\right)^2}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)} \beta\left(\frac{\gamma_1}{2}+2, \frac{\gamma_2}{2}-2\right) \qquad \beta(m,n = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$$

$$= \frac{\left(\frac{\nu_2}{\nu_1}\right)^2 \Gamma\left(\frac{\nu_1}{2}+2\right) \Gamma\left(\frac{\nu_2}{2}-2\right) \Gamma\left(\frac{\nu_1}{2}+\frac{\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right) \Gamma\left(\frac{\nu_1}{2}+2+\frac{\nu_2}{2}-2\right)}$$

$\Gamma(n+2) = (n+1)\Gamma(n+1)$

$\Gamma(n) = (n-1)\Gamma(n-1)$

$$= \frac{\left(\nu_2/\nu_1\right)^2 \Gamma\left(\frac{\nu_1}{2}+2\right) \Gamma\left(\frac{\nu_2}{2}-2\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)}$$

$$= \frac{\left(\nu_2/\nu_1\right)^2 \left(\frac{\nu_1}{2}+1\right) \Gamma\left(\frac{\nu_1}{2}+1\right) \Gamma\left(\frac{\nu_2}{2}-2\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)}$$

$\Gamma(n+1) = n\Gamma(n)$

$$= \frac{\left(\nu_2/\nu_1\right)^2 \left(\frac{\nu_1}{2}+1\right) \left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}-2\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)}$$

$$= \frac{\left(\nu_2^2/\nu_1^2\right) \left(\frac{\nu_1}{2}+1\right) \left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}-2\right)}{\left(\frac{\nu_2}{2}-1\right) \left(\frac{\nu_2}{2}-2\right) \Gamma\left(\frac{\nu_2}{2}-2\right)}$$

$\Gamma(n) = (n-1)\Gamma(n-1)$

$= (n-1)(n-2)\Gamma(n-2)$

$$\mu_2' = \frac{\left(\nu_2^2/\nu_1\right)\left(\frac{\nu_1}{2}+1\right)}{2\left(\frac{\nu_2}{2}-1\right)\left(\frac{\nu_2}{2}-2\right)}$$

lll'y on putting $r=3$ and $r=4$ we get μ_3' and μ_4' respectively.

From which central moments μ_2 & μ_4 can be obtained.

Applications of F distribution

2m

F distribution has the following application in statistical theory:

F test for equality of population variance whether two independent sample x_i, $i=1,2,\dots n$, y_i ; $i=1,2,\dots n$ have being drawn from the

normal population with the same variance σ^2;
whether the two independent estimate of population variance are homogeneous or not under the null hypothesis (H_0)

$$\sigma_x^2 = \sigma_y^2 = \sigma^2$$

∴ The population variance are equal,
Two independent estimates of the population variation are homogeneous. Thus the statistics F is given by $F = \frac{\sigma_x^2}{\sigma_y^2}$

where,

$$\sigma_x^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\sigma_y^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

are estimated unbiased of the common population variance obtained from the independent sample and it follows Snedecor's 'F' distribution ($\gamma_1, \gamma_2, \ldots$ degrees of freedom)
where

$$\gamma_1 = n_1 - 1$$
$$\gamma_2 = n_2 - 1 \quad \ldots \quad \gamma_n = n_n - 1$$

F-Test:

In the F-test we have two sample size & they are independent these samples are drawn from the normal population with the same variance (σ^2)

Procedure for Test of Equality of Two Population Variance:

Step 1 : State null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$

Step 2 : State alternative hypothesis $H_1: \sigma_1^2 \neq \sigma_2^2$

Step 3: Level of Significance $\alpha = 1\%$ (or) 5%.

Step 4 : The Test Statistics is

$$F = \frac{S_1^2}{S_2^2} \quad (F > 1)$$

where

$$S_1^2 = \frac{n_1}{n_1 - 1} s_1^2$$

$$S_2^2 = \frac{n_2}{n_2 - 1} s_2^2$$

where n_1, n_2 are the size's of samples drawn from the two populations.

s_1^2, s_2^2 are the sample variances.

In defining F if $F < 1$ then $F = \frac{S_2^2}{S_1^2}$

Step 5 : Degrees of freedom if $F > 1$ the no. of degrees of freedom $= (n_1 - 1, n_2 - 1)$

If $F < 1$ the ndf $= (n_2 - 1, n_1 - 1)$

Step 6 : Table value of F

Step 7 : If C.V < T.V then H_0 is accepted & H_1 is rejected otherwise H_0 is rejected, H_1 is accepted.

1) If two Samples have the following results $n_1 = 10$, $\Sigma(x_i - \bar{x})^2 = 90$, $n_2 = 12$, $\Sigma(y_i - \bar{y})^2 = 108$. Test whether the Sample came from the same variance of the population.

Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

Level of significance $\alpha = 5\%$.

Given: $n_1 = 10$, $n_2 = 12$

variance of 1st Sample is $s_1^2 = \frac{\Sigma(x_i - \bar{x})^2}{n_1}$

$$s_1^2 = \frac{90}{10} = 9$$

variance of 2nd Sample is

$$s_2^2 = \frac{\Sigma(y_i - \bar{y})^2}{n_2}$$

$$s_2^2 = \frac{108}{12} = 9$$

Test Statistic $F = \frac{S_1^2}{S_2^2}$

$$S_1^2 = \frac{n_1}{n_1 - 1} s_1^2$$

$$S_1^2 = \frac{10}{10-1} \times 9 = 10$$

$$S_2^2 = \frac{n_2}{n_2 - 1} s_2^2 = \frac{12}{12-1} \times 9 = 9.818$$

$$F = \frac{S_1^2}{S_2^2} = \frac{10}{9.818} = 1.01885 > 1$$

Degrees of freedom $\gamma = (n_1 - 1, n_2 - 1)$

$$\gamma = (10-1, 12-1)$$

$$\gamma = (9, 11)$$

Table value at 5% level of significance

table value of $(9, 11)$

Degrees of freedom is $F_\alpha = 2.90$

i.e., C.V < T.V

$1.01885 < 2.90$

∴ H_0 is accepted, H_1 is rejected.

∴ Samples came from the population with same variance.

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2) Time taken by workers performing a job are given below.

Sample 1	20	16	26	27	23	22	-
Sample 2	27	33	42	35	32	34	38

Test whether there is any significant different between the variances of time distribution.

$n_1 = 6$, $n_2 = 7$

Sample - I, Sample - II

x	$d = (x-\bar{x})$	$d^2 = (x-\bar{x})^2$	y	$d = (y-\bar{y})$	$d^2 = (y-\bar{y})^2$
20	-2.33	15.4289	27	-7.43	55.2049
16	-6.33	40.0689	33	-1.43	2.0449
26	3.67	13.4689	42	7.57	57.3049
27	4.67	21.8089	35	0.57	0.3249
23	0.67	0.4489	32	-2.43	5.9049
22	-0.33	0.1089	34	-0.43	0.1849
-	-	-	38	3.57	12.7149
$\Sigma x = 134$	$\Sigma d = 0.02$	$\Sigma d^2 = 81.334$	$\Sigma y = 241$	$\Sigma d = 0.01$	133.7143

$$\bar{X} = \frac{\Sigma x}{n_1} = \frac{134}{6} = 22.33$$

$$\bar{Y} = \frac{\Sigma Y}{n_2} = \frac{241}{7} = 34.43$$

variance $S_1^2 = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{21.3334}{6} - \left(\frac{9.02}{6}\right)^2}$

$S_1^2 = 3.6818$

$S_2^2 = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{133.7143}{7} - \left(\frac{-0.01}{7}\right)^2}$

$= 4.3705$

Test Statistics is

$F = \frac{S_1^2}{S_2^2}$

$S_1^2 = \frac{n_1 s_1^2}{n-1} = \frac{6(3.6818)}{6-4} = 4.418164$

$S_2^2 = \frac{n_2 s_2^2}{n_2-1} = \frac{7(4.3705)}{7-1} = 5.099$

$F = \frac{4.418164}{5.099} = 0.8664 < 1$

If $F < 1$ then $F = \frac{S_2^2}{S_1^2}$

$F = \frac{5.099}{4.41816} = 1.1541$

Degree of freedom $\nu = (n_2 - 1, n_1 - 1) = (7-1, 6-1)$

$= (6, 5)$

The table value at 5% for (6,5) degree of freedom is $F_\alpha = 4.95$

Calculated value is less than table value

$1.1541 < 4.95$

H_0 is accepted.

There is no significance difference between the variance of time distribution.

3) From the following data test the difference between the variance of significance at 5% level of significance.

Sum of square of deviation from mean:	84.4	102.6
Size	8	10
Sample	A	B

Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

Level of significance $\alpha = 5\%$

Given

$n_1 = 8, \quad n_2 = 10$

$\Sigma (x_i = \bar{x})^2 = 84.4$

$\Sigma (y_i - \bar{y})^2 = 102.6$

small s

$s_1^2 = \dfrac{\Sigma (x_i - \bar{x})^2}{n_1} = \dfrac{84.4}{8} = 10.55$

$s_2^2 = \dfrac{\Sigma (y_i - \bar{y})^2}{n_2} = \dfrac{102.6}{10} = 10.26$

Test Statistics $F = S_1^2 / S_2^2$

capital S

$S_1^2 = \dfrac{n_1 s_1^2}{n_1 - 1} = \dfrac{8 \times 10.55}{8-1} = 12.057$

$S_2^2 = \dfrac{n_2 s_2^2}{n_2 - 1} = \dfrac{10 \times 10.26}{10-1} = 11.4$

$F = \dfrac{S_1^2}{S_2^2} = \dfrac{12.05-7}{11.4}$

$F = 1.05764 > 1$

Degrees of freedom ~~of~~ $(n_1 - 1), (n_2 - 1) = (7, 9)$

The table value at 5%. for (7,9) degree of freedom $F_\alpha = 3.29$

$C.V < T.V$

Ho is accepted.

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4) In a sample of 8 observation, The sample of square deviation of item from the mean was 94.5. In other sample of 10 observation the value was found to be 101.7. Test whether the difference in variance is significant at 5% level.

Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

$\alpha = 5\%$

Given

$n_1 = 8$, $\Sigma(x-\bar{x})^2 = 94.5$

$n_2 = 10$, $\Sigma(y-\bar{y})^2 = 101.7$

$$s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1} = \frac{94.5}{8} = 11.8125$$

$$s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2} = \frac{101.7}{10} = 10.17$$

Test statistics is $F = S_1^2 / S_2^2$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{8 \times 11.8125}{8 - 1}$$

$$S_1^2 = 13.5$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{10 \times 10.17}{10-1} = 11.3$$

$$F = \frac{S_1^2}{S_2^2} = \frac{13.5}{11.3} = 1.194 > 1$$

Degrees of freedom $\gamma = (n_1 - 1, n_2 - 1) = (7,9)$

The table at 5% for (7,9) degrees of freedom $F_\alpha = 3.29$

$$1.194 < 3.29$$

H_0 is accepted.

Unit-4

A computer while calculating correlation coefficient between two variables x and y from 25 pairs of observations obtained the following results.

$n = 25, \Sigma x = 125, \Sigma x^2 = 650, \Sigma y = 100, \Sigma y^2 = 460,$

$\Sigma xy = 508$ it was however later discovered at the time of checking that he had copied wrong pairs as

x	y
6	14
8	6

while the correct values where

x	y
8	12
6	8

obtained the correct value of correlation coefficient.

Soln)

$$r(x,y) = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$cov(x,y) = \frac{1}{n}\left[\Sigma xy - \bar{x}\bar{y}\right]$$

correct Σx = wrong Σx - (Incorrect values) + (correct values)

$= 125 - 6 - 8 + 6 + 8$

$= 125$

Correct Σy = Wrong Σy - (Incorrect Value) + (correct value)

$= 100 - 14 - 6 + 12 + 18 = 100$

Σx^2 = wrong Σx^2 - (incorrect value)^2 + (correct value)

$\Sigma x^2 = 650 - (6-8)^2 + (8\times6)^2 = 650$

Correct Σy^2 = wrong Σy^2 - (incorrect value)^2 + (correct value)^2

$= 460 - (14\times6)^2 + (6\times8)^2 = 436$

Σxy = wrong Σxy - (incorrect value) + (correct Value)

$= 508 - (6\times14) - (8\times6) + (8\times12) + (6\times8)$

$= 520$

$\bar{x} = \frac{\Sigma x}{n} = \frac{125}{25} = 5$

$\bar{y} = \frac{\Sigma y}{n} = \frac{100}{25} = 4$

$\text{cov}(x.y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y} = \frac{1}{2}(520) - 20$

$= 20.8 - 20. = 0.8$

$\sigma_x^2 = \frac{1}{n}\Sigma x^2 - \bar{x}^2 = \frac{1}{25}(650) - 5^2 = 1$

$\sigma_y^2 = \frac{1}{n}\Sigma y^2 - \bar{y} = \frac{1}{25}(436) - 4^2 = 1.4$

$\sigma_y = 1.183$

$\therefore \frac{\text{cov}(x,y)}{\sigma_x\sigma_y} = \frac{0.8}{(1)(1.183)} = 0.67.$

Proof

$$F = \frac{S_x^2}{S_y^2} = \left[\frac{n_1}{n_1-1} S_x^2\right] \Big/ \left[\frac{n_2}{n_2-1} S_y^2\right]$$

$$= \left[\frac{n_1 S_x^2}{\sigma_x^2} \cdot \frac{1}{(n_1-1)}\right] \Big/ \left[\frac{n_2 S_y^2}{\sigma_y^2} \cdot \frac{1}{(n_2-1)}\right]$$

$(\because \sigma_x^2 = \sigma_y^2 = \sigma^2$, under $H_0)$

Since $\frac{n_1 S_x^2}{\sigma_x^2}$ and $\frac{n_2 S_y^2}{\sigma_y^2}$ are independent Chi-square variates with (n_1-1) and (n_2-1) d.f. respectively, F follows Snedecor's F-distribution with (n_1-1, n_2-1) d.f.

Example 16.25

Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins show that sample S.D of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the true variances are equal, against the alternative that they are not, at the 10% level. [Assume that $P(F_{10,8} \geq 3.35) =$ 0.05 and $P(F_{8,10} \geq 3.07) = 0.05$]

We want to test Null hypothesis,

$H_0 = \sigma_x^2 = \sigma_y^2$ against the

Alternative Hypothesis:

$H_1 : \sigma_x^2 \neq \sigma_y^2$ (Two-tailed) we are given: $n_1 = 11$, $n_2 = 9$, $S_X = 0.8$ and $S_Y = 0.5$

Under the null hypothesis, $H_0 : \sigma_X^2 = \sigma_Y^2$,

the statistic:

$F = \frac{S_X^2}{S_Y^2}$ follows F distribution with

$(n_1 - 1, n_2 - 1)$ d.f.

$n_1 s_X^2 = (n_1 - 1) S_X^2 \Rightarrow S_X^2 = \left(\frac{n_1}{n_1 - 1}\right) s_X^2 = \left(\frac{11}{10}\right) \times (0.8)^2$

$= 0.704$

Similarly,

$S_Y^2 = \left(\frac{n_2}{n_2 - 1}\right) s_Y^2 = \left(\frac{9}{8}\right) \times (0.5)^2 = 0.28125$

$\therefore F = \frac{0.704}{0.28125} = 2.5$

The significant values of F for two-tailed test at level of significance $\alpha = 0.10$ are:

$$\left.\begin{aligned} F &> F_{10,8}(\alpha/2) = F_{10,8}(0.05) \\ \text{and } F &< F_{10,8}(1 - \alpha/2) = F_{10,8}(0.95) \end{aligned}\right\} \rightarrow ①$$

we are given the tabulated (significant) values:

$P(F_{10,8} \geq 3.35) = 0.05 \Rightarrow F_{10,8}(0.05) = 3.35 \rightarrow ②$

Also $P(F_{8,10} \geq 3.07) = 0.05$

$\Rightarrow P\left(\frac{1}{F_{8,10}} \leq \frac{1}{3.07}\right) = 0.05$

$\Rightarrow P(F_{10,8} \leq 0.326) = 0.05 \Rightarrow P(F_{10,8} \geq 0.326) = 0.95$

$\rightarrow ③$

Hence from ①, ② and ③ the critical values for testing $H_0 : \sigma_X^2 = \sigma_Y^2$, against $H_1 : \sigma_X^2 \neq \sigma_Y^2$ at level of significance $\alpha = 0.10$ are given by: $F > 3.35$ and $F < 0.326 = 0.33$

Since, the calculated value of F $(= 2.5)$ lies between 0.33 and 3.35, it is not

Significant and hence null hypothesis of equality of population variances may be accepted at level of significance $\alpha = 0.10$

Example 16.27

Two random samples gave the following results:

Sample	Size	Sample mean	Sum of Squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% level of significance. [Given : $F_{0.05}(9,11) = 2.90$, $F_{0.05}(11,9) = 3.10$ (approx). and $t_{0.05}(20) = 2.086$, $t_{0.05}(22) = 2.07$]

A normal population has two parameters, mean μ and variance σ^2. To test if two independent samples have been drawn from the same normal population, we have to test (i) the equality of population means, and (ii) the equality of population variances.

Null Hypothesis : The two samples have been drawn from the same normal population

$H_0 : \mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$.

Equality of means will be tested by applying t-test and equality of variances will be tested by applying F-test. Since t-test assumes $\sigma_1^2 = \sigma_2^2$, we shall first apply F-test and t-test. In usual notations, we are given:

$n_1 = 10$, $n_2 = 12$; $\bar{x}_1 = 15$, $\bar{x}_2 = 14$,
$\Sigma(x_1 - \bar{x}_1)^2 = 90$, $\Sigma(x_2 - \bar{x}_2)^2 = 108$

F-test: Here

$$S_1^2 = \frac{1}{n_1 - 1} \Sigma(x_1 - \bar{x}_1)^2 = \frac{90}{9} = 0,$$

$$S_2^2 = \frac{1}{n_2 - 1} \Sigma(x_2 - \bar{x}_2^{\,2}) = \frac{108}{11} = 9.82$$

Since $S_1^2 > S_2^2$, under $H_0: \sigma_1^2 = \sigma_2^2$, the test Statistic is

$$F = \frac{S_1^2}{S_2^2} \sim F(n_1 - 1, n_2 - 1) = F(9, 11)$$

$$F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.018$$

Tabulated $F_{0.05}(9, 11) = 2.90$. Since calculated F is less than tabulated F, it is not significant. Hence null hypothesis of equality of population variances may be accepted.

Since $\sigma_1^2 = \sigma_2^2$, we can now apply t test for testing $H_0: \mu_1 = \mu_2$

t-test Under $H_0': \mu_1 = \mu_2$, against alternative hypothesis, $H_1': \mu_1 \neq \mu_2$, the test Statistics is:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{n_1 + n_2 - 2} = t_{20}$$

where $S^2 = \frac{1}{n_1 + n_2 - 2}\left[\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2\right]$

$$= \frac{1}{20}(90 + 108) = 9.9$$

$$\therefore t = \frac{15 - 14}{\sqrt{9.9\left(\frac{1}{10} + \frac{1}{12}\right)}} = \frac{1}{\sqrt{9.9 \times \frac{11}{60}}} = \frac{1}{\sqrt{1.815}} = 0.742$$

Tabulated $t_{0.05}$ for 20 d.f = Since $|t| < t_{0.05}$, it is not significant. Hence the hypothesis ~~$(H_0' : \sigma_1^2 = \sigma_2^2$ are$)$~~ $H_0' : \mu_1 = \mu_2$ may be accepted. Since both the hypothesis, ie., $H_0' : \mu_1 = \mu_2$ and $H_0 : \sigma_1^2 = \sigma_2^2$ are accepted, we may regard that the given samples have been drawn from the same normal population.

F-test for Testing the Significance of an Observed Multiple correlation coefficient.

If R is the observed multiple correlation coefficient of a variate with k other variates in a random sample of size n from a $(k+1)$ variate population, then Prof. R.A. Fisher proved that under the null hypothesis (H_0) that the multiple correlation coefficient in the population is zero, the statistic:

$$F = \frac{R^2}{1-R^2} \cdot \frac{n-k-1}{k}$$

conforms to F-distribution with $(k, n-k-1)$ d.f.

F-test for Testing the Significance of an Observed Sample correlation Ratio η_{yx}.

Under the null hypothesis that population correlation ratio is zero, the test statistic is:

$$F = \frac{\eta^2}{1-\eta^2} \cdot \frac{N-h}{h-1} \sim F(h-1, N-h)$$

where N is the size of the sample (from a bi-variate normal population) arranged in h arrays.

F-test for Testing the linearity of Regression.

For a sample of size N arranged in h arrays, from a bi-variate normal population, the test statistic for testing the hypothesis of linearity of regression is:

$$F = \frac{\eta^2 - r^2}{1-\eta^2} \cdot \frac{N-h}{h-2} \sim F(h-2, N-h)$$

F-test for Equality of Several Means.

This test is carried out by the technique of Analysis of variance, which plays a very important and fundamental role in Design of Experiments in Agricultural Statistics.

[For a detailed discussion of the Analysis of variance Technique, see Fundamental ~~role in Design of See F~~ of Applied Statistics by the same authors.]

Example 16.20

When $v_1 = 2$, show that the significance level of F corresponding to a significant probability P is: $F = \frac{v_2}{2}\left[P^{-(2/v_2)} - 1\right]$ where v_1 an v_2 have their usual meanings.

When $v_1 = 2$,

$$f(F) = \frac{1}{B\left(1, \frac{v_2}{2}\right)} \cdot \frac{2}{v_2} \frac{1}{\left(1+\frac{2}{v_2}F\right)^{(v_2/2)+1}}$$

$$= \frac{\Gamma\left(\frac{v_2}{2}+1\right)}{\Gamma 1 \, \Gamma \frac{v_2}{2}} \times \frac{2/v_2}{\left(\frac{2}{v_2}\right)^{(v_2/2)+1}\left(F+\frac{v_2}{2}\right)^{(v_2/2)+1}}$$

$$= \frac{\left(\frac{v_2}{2}\right)^{(v_2/2)+1}}{\left(F+\frac{v_2}{2}\right)^{(v_2/2)+1}}$$

Hence $P = \int_F^\infty f(F)\,dF = \left[\frac{v_2}{2}\right]^{(v_2/2)+1} \times \int_F^\infty \frac{dF}{\left(F+\frac{v_2}{2}\right)^{(v_2/2)+1}}$

$$= \left(\frac{v_2}{2}\right)^{(v_2/2)+1} \times \left| \frac{\left(F+\frac{v_2}{2}\right)^{-(v_2/2)}}{-\frac{v_2}{2}} \right|$$

$$= \left[\frac{\left(\frac{v_2}{2}\right)}{F+\frac{v_2}{2}}\right]^{v_2/2} = \frac{1}{\left(1+\frac{2}{v_2}F\right)^{v_2/2}}$$

$$P^{-(2/v_2)} = 1 + \frac{2F}{v_2}$$

$$F = \frac{v_2}{2}\left[P^{-(2/v_2)} - 1\right].$$

Example 16.21

X is a binomial variate with parameter n and p and F_{v_1, v_2} is an F-Statistic with v_1 and v_2 d.f. prove that:

$$P(X \le k-1) = P\left[F_{2k, 2(n-k+1)} > \frac{n-k+1}{k} \cdot \frac{p}{1-p}\right]$$

If $X \sim B(n, p)$, then we have

$$P(X \le k-1) = (n-k+1) \cdot \binom{n}{k-1} \int_0^q t^{n-k}(1-t)^{k-1}\, dt$$

$$= \frac{1}{\beta(n-k+1, k)} \int_0^q t^{n-k}(1-t)^{k-1}\, dt \rightarrow ①$$

$$P = P_r\left[F_{2k, 2(n-k+1)} > \frac{n-k+1}{k}\left(\frac{p}{1-p}\right)\right]$$

$$= \int_{\frac{n-k+1}{k} \cdot \frac{p}{q}}^{\infty} P\left[F_{2k, 2(n-k+1)}\right] dF$$

$$= \frac{1}{B(k, n-k+1)} \int_{\frac{n-k+1}{k} \cdot \frac{p}{q}}^{\infty} \frac{[k/(n-k+1)]^{k} \cdot F^{k-1}\, dF}{\left(1 + \frac{kF}{n-k+1}\right)^{n+1}} \rightarrow ②$$

put $1 + \frac{kF}{n-k+1} = \frac{1}{y}$

$\Rightarrow F = \frac{n-k+1}{k}\left(\frac{1-y}{y}\right)$ and $dF = \frac{n-k+1}{k} \cdot \frac{-dy}{y^2}$

$F = \infty \Rightarrow y = 0$ and $F = \frac{n-k+1}{k} \cdot \frac{p}{q}$

$\Rightarrow \frac{1}{y} = \frac{q+p}{q} = \frac{1}{q} \Rightarrow y = q$

Sub ②, we get:

$$P = \frac{1}{B(k, n-k+1)} \int_q^0 \left(1 - \frac{y}{y}\right)^{k-1} \cdot y^{n+1} \left(\frac{-dy}{y^2}\right)$$

$$= \frac{1}{B(k, n-k+1)} \int_0^q y^{n-k} (1-y)^{k-1}\, dy \to ③$$

From ①, ② and ③ we get the result.

Example 16.22

If $F(n_1, n_2)$ represents an F-variate with n_1 and n_2 d.f., prove that $F(n_2, n_1)$ is distributed as $1/F(n_1, n_2)$ variate. Deduce that

$$P[F(n_1, n_2) \geq c] = P[F(n_2, n_1) \leq \tfrac{1}{c}]$$

(or)

Show that how probability points of $F(n_2, n_1)$ can be obtained from those of $F(n_1, n_2)$

Let x and y be independent chi-square variates with n_1 and n_2 d.f. respectively. Then by definition, we have

$$F = \frac{(x/n_1)}{(y/n_2)} \sim F(n_1, n_2) \text{ and}$$

$$\frac{1}{F} = \frac{(y/n_2)}{(x/n_1)} \sim F(n_2, n_1) \to ①$$

Hence the result

we have:

$$P[F(n_1, n_2) \geq c] = P\left[\frac{1}{F(n_1, n_2)} \leq \frac{1}{c}\right] = P\left[F(n_2, n_1) \leq \frac{1}{c}\right] \to ①$$

Remark: Probability points of $F(n_2, n_1)$ from those of $F(n_1, n_2)$ Distribution.

Let $P[F(n_1,n_2) \geq c] = \alpha$

i.e. let c be the upper α-significant point of $F(n_1,n_2)$ distribution.

$$\therefore 1-\alpha = 1 - P[F(n_1,n_2) \geq c] = 1 - P\left[\frac{1}{F(n_1,n_2)} \leq \frac{1}{c}\right]$$

$$\alpha = P\left[F(n_2,n_1) \leq \frac{1}{c}\right] = 1 - P\left[F(n_2,n_1) \geq \frac{1}{c}\right]$$

From (1)

$$\therefore P\left[F(n_2,n_1) \geq \frac{1}{c}\right] = 1-\alpha$$

Thus $(1-\alpha)$ significant points of $F(n_2,n_1)$ distribution are the reciprocal of α-significant points of $F(n_1,n_2)$ distributions,

$$F_{8,9}(0.05) = 6.04$$

$$F_{4,8}(0.95) = \frac{1}{6.04}$$

Example 16.23

Prove that if $n_1 = n_2$, the median of F-distribution is at $F=1$ and that the quartiles Q_1 and Q_3 satisfy the condition $QQ_3 = 1$

Since $n_1 = n_2 = n$, (say)

The Median (M) of $F(n_1,n_2) = F(n,n)$ distribution is given by :

$$P[F(n,n) \leq M] = 0.5$$

$$P\left[\frac{1}{F(n,n)} \geq \frac{1}{M}\right] = 0.5 \Rightarrow P\left[F(n,n) \geq \frac{1}{M}\right] = 0.5$$

$$\left[\because \frac{1}{F(m,n)} = F(n,m)\right]$$

$$P\left[F(n,n) \le \frac{1}{M}\right] = 1 - P\left[F(n,n \ge \frac{1}{M}\right] = 1 - 0.5 = 0.5 \rightarrow (2)$$

From (1) and (2), we get

$$M = \frac{1}{M} \Rightarrow M^2 = 1 \text{ or } M = 1$$, the negative

value $M = -1$, is discarded since $F > 0$.

Hence the median of $F(n,n)$ distribution is at

$F = 1$.

Similarly, by definition of Q_1 and Q_3, we have

$$P[F(n,n) \le Q_1] = 0.25 \rightarrow (3)$$

and $P[F(n,n) \ge Q_3] = 0.25 \Rightarrow P\left[\frac{1}{F(n,n)} \le \frac{1}{Q_3}\right] = 0.25$

$$\Rightarrow P\left[F(n,n) \le \frac{1}{Q_3}\right] = 0.25 \quad \left[\because \frac{1}{F(m,n)} = F(n,m)\right] \rightarrow (4)$$

From (3) and (4), we get

$$Q_1 = \frac{1}{Q_3} \Rightarrow Q_1 Q_3 = 1$$

Example 16.24

Let $x_1, x_2, x_3, \dots x_n$ be a random sample from $N(0,1)$.

Define $\bar{x}_K = \frac{1}{K}\sum_{1}^{K} x_i$ and $\bar{x}_{n-K} = \frac{1}{n-K}\sum_{K+1}^{n} x_i$.

Find the distribution of:

a) $\frac{1}{2}(\bar{x}_K + \bar{x}_{n-K})$ b) $K\bar{x}_K^2 + (n-K)\bar{x}_{n-K}^2$

c) x_1^2/x_2^2 and d) x_1/x_2

a) Since $x_1, x_2, \dots x_n$ is a random sample from $N(0,1)$,

$\cancel{\bar{x}_K = \frac{1}{K}\sum_{1}^{K} x}$ $\bar{x}_K \sim N(0, \frac{1}{K})$ and

$$\bar{x}_{n-K} \sim N\left(0, \frac{1}{n-K}\right) \rightarrow (1)$$

Further, since $(x_1, x_2, \ldots, x_k)$ and $(x_{k+1}, x_{k+2}, \ldots, x_n)$ are independent, $\bar{x}_k$ and $\bar{x}_{n-k}$ are independent.

Hence,

$$\frac{1}{2}(\bar{x}_k + \bar{x}_{n-k}) = \frac{1}{2}\bar{x}_k + \frac{1}{2}\bar{x}_{n-k} \sim N\left\{0, \frac{1}{4k} + \frac{1}{4(n-k)}\right\}$$

$$\Rightarrow \frac{1}{2}(\bar{x}_k + \bar{x}_{n-k}) \sim N\left\{0, \frac{n}{4k(n-k)}\right\}$$

(b) from ①, we get

$$\frac{\bar{x}}{\sqrt{1/k}} \sim N(0,1) \text{ and } \frac{\bar{x}_{n-k}}{\sqrt{1/(n-k)}} \sim N(0,1)$$

$$k\bar{x}_k^2 \sim \chi^2_{(1)} \text{ and } (n-k)\bar{x}^2_{n-k} \sim \chi^2_{(1)}$$

Since $\bar{x}_k$ and $\bar{x}_{n-k}$ are independent, by additive property of chi-square distribution,

$$k\bar{x}_k^2 + (n-k)\bar{x}^2_{n-k} \sim \chi^2_{(1+1)} = \chi^2_{(2)}$$

c) Since $X_1 \sim N(0,1)$ and $X_2 \sim N(0,1)$ are independent, $X_1^2 \sim \chi^2_{(1)}$ and $X_2^2 \sim \chi^2_{(1)}$, are also independent.

Hence by definition of F-statistic,

$$\frac{X_1^2/1}{X_2^2/1} \sim F_{(1,1)} \Rightarrow \frac{X_1^2}{X_2^2} \sim F_{(1,1)}$$

d) X_1/X_2, being the ratio of two independent standard normal variates is a standard Cauchy variate.

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UNIT – III

2 MARKS

1. Define F distribution.
2. Write F testing the Linearity of Regression.
3. Describe F test for Equality of several Means.
4. F test for testing the significance of an observed Multiple Correlation Coefficient.

5 MARKS

1. Describe about Constant of F distribution
2. Write the Procedure for test of equality of two population variance
3. Explain Mode and Points of inflexion of F distribution.
4. Explain the Application of F distribution

10 MARKS

1. Derive of F distribution.
2. Pumpkins were grown under two experimental conditions.Two random samples of 11 and 9 pumpkins show the sample standard deviations of their weights as 0.8 and 0.5. Assuming that the weight distributions are normal. Test the hypothesis that the true variances are equal, against the alternative that they are not, at the 10% level.

REFERENE BOOKS

1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Co, New Delhi, Reprint 2019. Ch 16 (Sec16.1-16.3.3).

2. Vittal,P.R.(2004).Mathematicalstatistics.MarghamPublications.

3. Kapur, J. N &Saxena, H. C. (2010). Mathematical statistics (20thed.). S. Chand & Company Ltd.