Introduction :

Insurance means a production against natural was first consume by the adventures travelers of the sea who carried goods of value of faraway, places brawing all the periods of the sea ~~is~~ anticipation, and some profit in the trade by way of Insurance saved money families

Life Assurance : (in 2 unit) (assurance definition)

principles of life assurance :

life assurances provides finanial assistant to the dependent of the life assured in the event of his death. life assurance also provide finanical security in the old age after there ~~required~~ retired from an occupation and the income has been stop from his source.

Premium (cost of assurance benefit)

The payment of the cost can be alway arranged according to the conviniens of the person seeking the benefit the periodical payment made by life assure are called premium.

Single premium

If the life assured pay the full cost of the benefit straight way in which case it is called a single premium

Annual Premium

The cost of benefit we paid in equal yearly installment for life it is a benefit payable at the time of death. This installment are called Annual premium.

Endowment Assurance:

It is combaination of temporary assurance and pure Endowment Assurance, sum assured the payable on death of the life assure aged x. at entry during is some of 'n' years or on his survival at the end of 'n' years.

The present value of a assurance of 1 under this plan is denoted by a symbol.

$$A_{x:\overline{n}|} = \frac{1}{l_x}\{(v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots + v^n d_{x+n-1}) + v^n l_{x+n}\} \rightarrow (1)$$

$$\Rightarrow A_{x:\overline{n}|} = \frac{1}{l_x}\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots + v^n d_{x+n-1}\} + \frac{1}{l_x} v^n l_{x+n}$$

$$\Rightarrow A_{x:\overline{n}|} = \underset{\text{Temporary}}{A^1_{x:\overline{n}|}} + A_{x:\overset{1}{\overline{n}|}} \text{ pure Endowment}$$

Pure Endowment assurance:

under this assurance the sum is payable only if a person survivals a ~~defined~~ definite period. The present value of pure Endowment assurance of 1 payable to (x)

on his survival to the term assurance of `n' year is denoted by the Symbol

$A_{x:\overset{1}{\overline{n|}}}$

out of l_x lives l_{x+n} lives surive for `n' years (i.e) upto aged $x+n$

~~making~~ taking the discounting factor for `n' years we have

$$A_{x:\overset{1}{\overline{n|}}} = v^n \times \frac{l_{x+n}}{l_x} \rightarrow (2)$$

Double Endowment policy Assurance:

under this assurance the benefit payable on death of life assure aged x at entry during the term of `n' years is the basic Sum assure, where as the benefit payable on his Survival at the end of `n' years is double the basis Sum assure.

The expressance for the ~~basic~~ present Value of this assurance for a basic sum assured of 1 is given by

$$= A_{\overset{1}{x}:\overline{n|}} + 2\, A_{x:\overset{1}{\overline{n|}}}$$

$$= A_{\overset{1}{x}:\overline{n|}} + A_{x:\overset{1}{\overline{n|}}} + A_{x:\overset{1}{\overline{n|}}}$$

$$= A_{x:\overline{n|}} + A_{x:\overset{1}{\overline{n|}}}$$

Fixed term Endowment (Marriage Endowment)

under this assurance the benefit is payable only at end of term selected, whether the life assure is alive or not.

The present value of Rupees 1 payable at the end of 'n' year is v^n.

The present value of this benefit is denoted by $A_{\overline{n|}} = v^n$

commutation functions big calculation reducting proper D_x, C_x, M_x and R_x:

⇒ Commutation function D_x

$$D_x = v^x \times l_x$$

⇒ Commutation function C_x

$$C_x = v^{x+1} \times d_x$$

⇒ Commutation function M_x

The commutation function M_x is obtained by summing the value of C_x from age x. upto the limiting age of the table. In gendrel

$M_x = C_x + C_{x+1} + C_{x+2} + \ldots$ to the limiting age to the table.

$$\therefore M_x = C_x + M_{x+1}$$

⇒ Commutation function R_x

R_x is obtained by summing the values of M_x from age x up to the limiting age of the table.

d - death

In general

$$R_x = M_x + M_{x+1} + M_{x+2} + \dots$$ to the limiting age of the table

$$\therefore R_x = M_x + R_{x+1}$$

Expression for present values of assurances benefit in terms of commutation function

=> Temporary assurances

consider $A^{1}_{x:\overline{n}|} = \frac{1}{l_x}\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots + v^n d_{x+n-1}\}$

multiply and divided by v^x

$$A^{1}_{x:\overline{n}|} = \frac{v^x}{v^x l_x}\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots + v^n d_{x+n-1}\}$$

$$A^{1}_{x:\overline{n}|} = \frac{1}{v^x l_x}\{v^{x+1} d_x + v^{x+2} d_x + v^{x+3} d_{x+2} + \dots + v^{x+n} d_{x+n-1}\}$$

$$= \frac{1}{D_x}\{C_x + C_{x+1} + C_{x+2} + \dots + C_{x+n-1}\}$$

$$= \frac{1}{D_x}\{(C_x + C_{x+1} + C_{x+2} + \dots) - (C_{x+n} + C_{x+n+1} + \dots)\}$$

$$A^{1}_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x}$$

=> whole life assurance

consider,

$$A_x = \frac{1}{l_x}\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots$$

$$= \frac{v^x}{v^x l_x}\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots$$

$$A_x = \frac{1}{v^x l_x}\left\{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + \dots\right\}$$

$$= \frac{1}{D_x}\left\{C_x + C_{x+1} + C_{x+2} + \dots\right\}$$

$$= \frac{1}{D_x}\left\{C_x + C_{x+1} + C_{x+2} + \dots\right\}$$

$$A_x = \frac{M_x}{D_x}$$

⇒ Pure Endowment

$$A_{x:\overset{1}{\overline{n|}}} = v^n \times \frac{l_{x+n}}{l_x}$$

$$= \frac{v^x v^n l_{x+n}}{v^x l_x}$$

$$= \frac{v^{x+n} l_{x+n}}{v^x l_x}$$

$$A_{x:\overset{1}{\overline{n|}}} = \frac{D_{x+n}}{D_x}$$

⇒ Endowment assurance:

$$A_{x:\overline{n|}} = \frac{1}{l_x}\left\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots + v^n d_{x+n-1} + v^n l_{x+n}\right\}$$

Multiply and divided v^x

$$A_{x:\overline{n|}} = \frac{v^x}{v^x l_x}\left\{v d_x + v^2 d_{x+1} + v^3 d_{x+2} + \dots + v^n d_{x+n-1} + v^n d_{x+n}\right\}$$

$$A_{x:\overline{n|}} = \frac{1}{v^x l_x}\left\{v^{x+1} d_x + v^{x+2} d_{x+1} + v^{x+3} d_{x+2} + \dots + v^{x+n} d_{x+n} + v^{x+n} l_{x+n}\right\}$$

$$= \frac{1}{D_x}\left\{(C_x + C_{x+1} + C_{x+2} + \dots + C_{x+n-1}) + D_{x+n}\right\}$$

$$= \frac{1}{D_x}\{M_x - M_{x+n} + D_{x+n}\}$$

$$A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$$

⇒ Double Endowments assurances:

$$= A_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1}$$

$$= \frac{M_x - M_{x+n} + D_{x+n}}{D_x} + \frac{D_{x+n}}{D_x}$$

$$= \frac{M_x - M_{x+n} + 2D_{x+n}}{D_x}$$

Increasing Temporary Assurance:

$$(IA)^1_{x:\overline{n}|} = \frac{1}{l_x}\left[vd_x + 2v^2 d_{x+1} + 3v^3 d_{x+2} + \ldots + nv^n d_{x+n-1}\right]$$

nultiple and divided by v^x

$$(IA)^1_{x:\overline{n}|} = \frac{v^x}{v^x l_x}\left[vd_x + 2v^2 d_{x+1} + 3v^3 d_{x+2} + \ldots + nv^n d_{x+n-1}\right]$$

$$(IA)^1_{x:\overline{n}|} = \frac{1}{v^x l_x}\left[v^{x+1} d_x + 2v^{x+2} d_{x+1} + 3v^{x+3} d_{x+2} + \ldots + nv^{x+n} d_{x+n-1}\right]$$

$$= \frac{1}{D_x}\left[C_x + 2C_{x+1} + 3C_{x+2} + \ldots + nC_{x+n-1}\right]$$

$$= \frac{1}{D_x}\left[\{C_x + 2C_{x+1} + 3C_{x+1} + \ldots + nC_{x+n-1} + (n+1)C_{x+n} + \ldots\} - \{(n+1)C_{x+n} + (n+2)C_{x+n+1} + \ldots\}\right]$$

$$nC_{x+n} + C_{x+n} + nC_{x+n+1} + 2C_{x+n+1} + \ldots$$

n is common

$$= \frac{1}{D_x}\left[R_x - n\{C_{x+n} + C_{x+n+1} + C_{x+n+2} + \ldots\} - \{C_{x+n} + 2C_{x+n+1} + 3C_{x+n+2} + \ldots\}\right]$$

$$= \frac{1}{D_x}\left[R_x - nM_{x+n} - R_{x+n}\right]$$

$$= \frac{R_x - R_{x+n} - nM_{x+n}}{D_x}$$

Increasing whole Assurance:

$$(IA)_x = \frac{1}{l_x}\{vd_x + 2v^2 d_{x+1} + 3v^3 d_{x+2} + \dots\}$$

$$= \frac{v^x}{v^x l_x}\{vd_x + 2v^2 d_{x+1} + 3v^3 d_{x+2} + \dots\}$$

$$= \frac{1}{v^x l_x}\{v^{x+1} d_x + 2v^{x+2} d_{x+1} + 3v^{x+3} d_{x+2} + \dots\}$$

$$= \frac{1}{D_x}\{C_x + 2C_{x+1} + 3C_{x+2} + \dots\}$$

$$= \frac{R_x}{D_x}$$

The following particular are given

x	25	26	27	28	29	30
l_x	97380	97088	96794	96496	96194	95887
d_x	292	294	298	302	307	313

calculate ignoring interest and expenses

i) The value of temporary assurance of ₹ 1000 for 2 years for a person aged 25

ii) The value of Endowments assurance benefit of ₹ 1000 for 4 years to a person aged 25

iii) The value of a ~~tempo~~ pure Endowment of ₹ 600 for a person aged 27 receivable on attaining age 30.

Soln:

i) The value of a Temporary assurance of ₹ 1000 for 2 years for a person age 25 is given by

General formula:

$$A^1_{x:\overline{n}|} = \frac{1}{l_x}\left\{v d_x + v^2 d_{x+1} + \dots + v^n d_{x+n-1}\right\}$$

ignore interest (ie) 'v'

$$A^1_{x:\overline{n}|} = \frac{1}{l_x}\left\{d_x + \cancel{v^2} d_{x+1} + \dots + d_{x+n-1}\right\}$$

$n = 2$
$x = 25$

$$A^1_{25:\overline{2}|} = \frac{1}{l_{25}}\left[d_{25} + d_{26}\right]$$

$$1000 A^1_{x:\overline{n}|} = (1000)\left[\frac{d_{25}+d_{26}}{l_{25}}\right]$$

$$1000 A^1_{25:\overline{2}|} = 1000\left[\frac{292+294}{97380}\right]$$

$$= 1000\left[\frac{586}{97380}\right]$$

$$= 1000\left[0.006\right]$$

$$= \text{RS. } 6.02$$

ii) The value of Endowment assurance benefit of ₹ 1000 for 4 years to a person age 25

$$A_{x:\overline{n}|} = \frac{1}{l_x}\left\{v d_x + v^2 d_{x+1} + \dots + v^n d_{x+n-1} + v^n l_{x+n}\right\}$$

$x = 25$
$n = 4$

$$1000 A_{25:\overline{4}|} = 1000\left[\frac{d_{25}+d_{26}+d_{27}+d_{28}+l_{29}}{l_{25}}\right]$$

$$= 1000\left[\frac{292+294+298+302+96194}{97380}\right]$$

$$= 1000\left[\frac{97380}{97380}\right]$$

$= 1000 [1]$

$= 1000$

iii) The value of a pure Endowment of ₹ 600 for a person aged 27 receivable on attaining age 30.

$$A_{x:\overset{1}{\overline{n|}}} = v^n \frac{l_{x+n}}{l_x}$$

$$600 A_{27:\overset{1}{\overline{3|}}} = 600 \times \frac{l_{30}}{l_{27}}$$

$$= 600 \times \left(\frac{95887}{96794}\right)$$

$$= 600 \times 0.9906$$

$$= 594.377$$

year

Net Premium for assurance plans:

⇒ Natural premiums based on our amount

It may not be possible for all persons desirous having insurance production to pay single premium

(i.e) A lump sum contribution, to secure benefits under an assurance plan

⇒ If the period of insurance is more than one year, it is possible to pay the premiums every year at the being of each year instruced of single lump sum payment.

⇒ level annual premium uniform (same value should pay)

The system of charging natural premiums can be replaced by a level annual premium system. where in persons taking insurance policys pay uniform premiums throughout the term of insurance.

Advantages of level annual premium (required the payment)

The level annual premium method as the following two advantages.

⇒ That younger age person can pay a little more than required This additional payment along with interest accumulated over a period of time go a long way to meet the cost of time benefits at higher ages.

⇒ It is easy for most of persons to make provisions for payment on uniform amount every year from their Earnings.

Symbols for level annual premium under various assurances plans:

Plans of assurance Age x, Term n	Symbol used to indicate	
	Single Premium	level annual premium
Term assurance	$A^1_{x:\overline{n}\vert}$	$P^1_{x:\overline{n}\vert}$
Pure Endowment assurance	$A_{x:\overset{1}{\overline{n}\vert}}$	$P_{x:\overset{1}{\overline{n}\vert}}$
Endowment assurance	$A_{x:\overline{n}\vert}$	$P_{x:\overline{n}\vert}$
whole life assurance	A_x	P_x

Mathematical for level annual premium under various plans for sum assured for Rs. 1

Temporary Assurance:

under this plan the life assured aged x will pay the level annual premium $P^1_{x:\overline{n}\vert}$ at the beging of each policy year for 'n' years.

The payment of premium cases from the policy anviresary following date of his death.

Thus level annual Premium $P^1_{x:\overline{n}\vert}$ form a temporary life anuity due

The Present value of level annual premium is $P^1_{x:\overline{n}\vert}\ \ddot{a}_{x:\overline{n}\vert}$

The present value of benefits = $A^1_{x:\overline{n}\vert}$

$$\therefore P^1_{x:\overline{n}\vert}\ \ddot{a}_{x:\overline{n}\vert} = A^1_{x:\overline{n}\vert}$$

$$P'_{x:\overline{n}|} = \frac{A'_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

$P'_{x:\overline{n}|}$ → Present value of level annual premium.

$A'_{x:\overline{n}|}$ → Present value of Term assurance

$\ddot{a}_{x:\overline{n}|}$ → Present value of annuity due

Expressing $A'_{x:\overline{n}|}$ and $\ddot{a}_{x:\overline{n}|}$ in terms of commutation function

$$P'_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x} \div \frac{N_x - N_{x+n}}{D_x}$$

$$P'_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}$$

Problem:

Find the expected number of death and total expected claim amount payable for the following a group of 10,000 persons all aged 35 seeking to provide an amount of ₹ 10,000 to their family In case of death during the next 10 years.

Age x	35	36	37	38	39	40	41	42	43	44
l_x	10,000	9,972	9,941	9,907	9,869	9,827	9,780	9,728	9671	9608
d_x	28	31	34	38	42	47	52	57	63	70

Soln:

⇒ Total expected No. of death among 10,000 persons all aged 35 before the ~~attend~~ attain aged 45.

$$= \sum_{35}^{44} d_x$$

$$= d_{35} + d_{36} + \dots d_{44}$$

$$= 28 + 31 + 34 + 38 + 42 + 47 + 52 + 57 + 63 + 70$$

=462

$\boxed{= 462}$

Total expected claim amount payable

=462 X10000

= 4620000

2. In the above example consider the insurer earns ~~interse~~ interest at 6% p.a find the Present value of claims payable in the year 'n'.

Same Question value

Soln:

Age x	l_x	d_x	PV of Claims payable in 'n' year 'PV' = $d_x \times v^n \times 10000$	Policy year 'n'	$v = \frac{1}{1+i} = \frac{1}{1+0.06}$ $v^n = \frac{1}{1.06} = 0.9433$
35	10,000	28	$d_{35} \times v^1 \times 10{,}000$ $28 \times 0.9433 \times 10{,}000$ = 264124	1	0.9433
36	9,972	31	$31 \times 0.8898 \times 10{,}000$ = 2,75,838	2	0.8898
37	9,941	34	$34 \times 0.83936 \times 10{,}000$ = 285,382	3	0.83936
38	9,907	38	$= 38 \times 0.7917 \times 10{,}000$ = 300,846	4	~~0.4962~~ 0.7917
39	9,869	42	$= 42 \times 0.7468 \times 10{,}000$ = 313,656	5	0.7468
40	9,827	47	$= 47 \times 0.7045 \times 10{,}000$ = 331,115	6	0.7045
41	9,780	52	$= 52 \times 0.6645 \times 10{,}000$ = 345540	7	0.6645
42	9,728	57	$= 57 \times 0.6269 \times 10{,}000$ = 357,333	8	0.6269
43	9,671	63	$= 63 \times 0.5913 \times 10{,}000$ = 372,519	9	0.5913
44	9,608	70	$= 70 \times 0.5578 \times 10{,}000$ = 390,460	10	0.5578

3236813

This amount is Shared by 10,000 persons

Thus single premium chargable to each person works out to

$$= \cancel{8}^{3}236,813$$

$$= \frac{3236,813}{10,000}$$

$$= 323.6813$$

3. In the above example find the natural premium and level annual premium by temporary assurances.

Soln:

x	l_x	d_x	Natural premium $= \frac{d_x}{l_x} \times 10,000 \to$ future value
35	10,000	28	$\frac{28}{10,000} \times 10000$ $= 28$
36	9,972	31	$= 31.09 = \frac{31}{9972} \times 10000$
37	9,941	34	$= 34.20 = \frac{34}{9941} \times 10000$
38	9,907	38	$= 38.35$
39	9,869	42	$= 42.56$
40	9,827	47	$= 47.83$
41	9,780	52	$= 53.167$
42	9,728	57	$= 58.59$
43	9,671	63	$= 65.14$
44	9,608	70	$= 72.86$

Level annual premium by temporary assurance for 10 years on the life of a aged 35 (ignoring interest)

In first year 10,000 person paying Premium

$\therefore$ In first year total premium received

$= 10,000P$

$\Rightarrow$ Similarly in second year total premium recived $= 9972P$

Third year total premium recived

$= 9941P$

$\Rightarrow$ Ten th year total premium recived

$= 9608P$

$\Rightarrow$ In Ten years total premium recived will be sum of yearly premium

$= 10,000 + 9972 + 9941 + 9907 + 9869 + 9827 + 9780 + 9728 + 9671 + 9608$

$= 98,303P$

Total claim amount $= 462 \times 10,000$

$= 46,20,000$

$\therefore 98303P = 46,20,000$

$$P = \frac{46,20,000}{98303}$$

$P = 46.99$

$\boxed{P = Rs.\ 47}$

level annual premium.
(adding interest)

level annual premium taking into an interest at 6%.

The present value of claims paid during 10 years. works out to RS 323.6813

Let level annual premium is 'P'

$V = \frac{1}{1+i}$, $i = 0.06$

$V = \frac{1}{1+0.06}$

$= \frac{1}{1.06}$

$= 0.9433$

The present value of 10,000 P recived at being at first year is = 10,000P

The present value of 9972P recived at being of second year is $= (9972P) \times V$

$= (9972P)(0.9433)$

$= 9406.58P$

$= 9407P$

The present value of 9941P recived at being of Third year is $= (9941P) \times V^2$

$= (9941P)(0.9433)^2$

$= (9941P)(0.8898)$

$= 8845.50P$

$= 8846P$

The present value of 9907P recived at being of fourth year is $= (9907P) \times V^3$

$= (9907P)(0.8393)$

$= 8314.94P$

$= 8315 P$

The present value of 9869 P recived at being of fifth year is $= (\cancel{968}\ 9869P) \times V^4$

$= (9869P) \times 0.7917$

$= 7813.28\ P$

$= 7813\ P$

The present value of 9827 P recived at being of sixth year is $= 9827\ P \times V^5$

$= 9827\ P \times 0.7468$

$= 7338.80\ P$

$= 7339\ P$

The present value of 9780 P recived at being of ~~sixth~~ seventh year is $= 9780\ P \times V^6$

$= 9780\ P \times 0.7045$

$= 6890.01\ P$

$= 6890 P$

The present value of 9728 P received at being of Eight year is $= 9728\ P \times V^7$

$= 9728\ P \times 0.6645$

$= 6464.25 P$

$= 6464 P$

The present value of 9671 P received at being of Nine year is $= 9671 P \times V^8$

$= 9671 P \times 0.6269$

$= 6062.74 P$

$= \cancel{6068}\ 6063\ P$

The present value of 9608P received at being of Tenth year is $= 9608P \times V^{9}$

$= 9608P \times 0.5913$

$= 5681.21P$

$= 5681P$

Total of present value of premium

$= 76{,}818P$

$\therefore 76818P = 3236813$

$P = \frac{32{,}36{,}813}{76818}$

$P = 42.13$

$P = 42$

$\boxed{P = Rs.42}$