MARUDHAR KESARI JAIN COLLEGE FOR WOMEN (AUTONOMOUS)

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1st M.COM – Semester - I

E-Notes (Study Material)

Allied Paper -1: Business Mathematics and Operation Research – I **Code: 24PCOE12**
Unit: 2 –Probability Distribution **Theory of Probability- Probability rules – Baye's theorem(Proof only) – Probability Distribution – Characteristics and application of Binomial, Poisson and Normal distribution.. (15 Hours)**
Learning Objectives: To understand the Probability Distribution
Course Outcome: Explain the Characteristics and application of probability

Overview:

In Statistics, the **probability distribution** gives the possibility of each outcome of a random experiment or event. It provides the probabilities of different possible occurrences. Also read, events in probability here.

To recall, the **probability is a measure of uncertainty of various phenomena**. Like, if you throw a dice, the possible outcomes of it, is defined by the probability. This distribution could be defined with any random experiments, whose outcome is not sure or could not be predicted. Let us discuss now its definition, function, formula and its types here, along with how to create a table of probability based on random variables.

Probability is the likelihood that an event will occur, an event being a specific outcome of an experiment. For example, if there is a typical deck of playing cards, and we wish to know how likely it is to draw an ace, the specific outcome, or event, is drawing an ace and we would calculate the probability of ace.

- Probabiity Distribution
- Probability rules
- Baye's Theorem
- Binomial Distribution
- Normal Distribution
- Poisson Distribution

UNIT-2 Probability Distribution

Random Experiment or trial:

An action (or) an operation which can produce any result or outcome is called a random experiment or a trial.

Eg. Tossing a coin is an experiment or trail, when you toss, it falls heads up or tail up.

Events:

Any possible outcome of an experiment is called an Event.

Eg - (H,T) (T,H)

Impossible Event:

An event can never occur when a certain random experiment is called impossible event.

Simple event:

An event is called simple if it correspond to a single possible outcome.

Compound Event:

When two or more event take place simultaneously where occurance is known as compound event.

Eg:- Throwing a die.

Favourable event:-

The number of cases favourable to an event in a trial is a number of outcome which entail the happening of the event.

Eg:- In throwing of two dias the number of cases favourable to getting three is {(1,2)(2,1)}

Mutually exclusive event:-

If two or more events cannot occur simultaneously in trial of an experiment, then such events are mutually exclusive events (or) disjoint events.

Eg: Events which cannot occur together or simutaneously.

A coin is tossed, either head or tail can be up but both cannot be up at the same time.

x. Exhaustive Event:-

The total number of possible outcome of a random experiment is called exhaustive event.

Eg- Tossing a coin, the possible outcomes are Head or Tail.

Equally likely event:-

Two or more events are said to be equally likely if each has an equal chance to occur.

Eg:- In a throw of die, the coming up of 1,2,3,4,5,6 is equally likely.

Independent Event:-

Two events are said to be independent if the occurance of the event does not affect the occurance of the other.

Dependent Event:

Two events are said to be dependent if the occurance of one event affect the probability of the others.

Sample Space:

The set of all possible outcome is known as sample space.

Eg: When we throw a die the possible outcomes are

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

probability:

The probability for the occurance of an event is defined as the ratio between the number of favourable outcomes for the occurance of the event and the total number of possible outcome.

$$\left.\begin{matrix}\text{probability of an} \\ \text{event}\end{matrix}\right\} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcome}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

Relative frequency Approach:

The probability of an event A is the ratio of the number of times that A has occured in n trails of an experiment

$$P(A) = \frac{n(S)}{n} = \frac{\text{Total number of outcome}}{n}$$

Axioms of probability:

The axioms of probability are

i) $P(A) \geq 0$

ii) $P(S) = 1$

iii) If A and B are mutually exclusive event then $P(A \cup B) = P(A) + P(B)$

(X) Fundamental Rules of probability :-

Let S be the sample space of an experiment that is partitioned into mutually exclusive and exhaustive event $A_1, A_2 \dots A_n$ which may be elementary or compound.

The probability of any event A in S is governed by the following rules.

i) Each probability should fall between 0 and 1 (i.e) $0 \leq P(A_i) \leq 1$ for all 'i'.

ii) The probability of event A_i, In otherwords, the probability of an event is restricted to the range zero to one inclusive, where zero represents an impossible event and one represents a certain event.

Eg:- probability of the number seven occuring, on rolling a dice, $P(7)=0$, because this number is an impossible event for this experiment.

iii) $P(S) = P(A_1) + P(A_2) + \dots + P(A_n) = 1$ where $P(S)$ is the probability of the certain event.

The sum of probabilities of all simple events constituting the sample space is equal to one.

Similarly, the probability of an impossible event or an empty set is zero (i.e) $P(\phi)=0$

iv) If events A_1 and A_2 are two elements in S and if occurence of A_1 implies that A_2 occurs, that is., if A_1 is a subset of A_2, then the probability of A_1 is less than or equal to the probability of A_2

i.e $P(A_1) \leq P(A_2)$

v) $p(\bar{A}) = 1 - p(A)$, the probability of an event that does not occur is equal to one minus the probability of the event that does occur.

Probability Terms:

If A and B are two events then

$A \cup B$ = an event which represents the occurrence of either A or B or both.

$A \cap B$ = an event which represents the simultaneous occurrance of A and B

$\bar{A}$ = Complement of A and represents non occurrence of A.

$\bar{A} \cap \bar{B}$ = Both A & B do not occur.

$\bar{A} \cap B$ = event A does not occur but event B occurs.

$A \cap \bar{B}$ = event A occurs but event B does not occure.

$(A \cap \bar{B}) \cup (\bar{A} \cap B)$ = Exactly one of the two events A & B occurs.

Problems.

atmost one hand (1 or 0) not more than

① A coin is tossed twice. Find the probability of getting atleast one head.

The possible outcome when a coin is tossed twice

$= \{HH, TT, HT, TH\}$

Total number of possible outcomes $= 4$

The favourable outcome for the outcome atleast one head are

$\{HH, HT, TH\}$

Number of favourable outcomes $= 3$

probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$$P(A) = \frac{3}{4}\ \frac{n(A)}{n(S)}$$

The probability of getting atleast one head $3/4$

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2 Two coins are tossed simultaneously. What it is the probability of getting a head and a tail.

The possible outcomes are {HH, HT, TH, TT}

No. of. possible outcomes = 4.

∴ Favourable outcomes for the event getting a head and tail is {HH}, {TT}

No. of. favourable outcomes = 2

The probability of getting a head and a tail

$2/4 = 1/2$

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3. Three coins are tossed. Find the probability of getting a) Atleast one head b) Exactly two heads.

The possible outcomes are {(HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)}

No. of. possible outcomes = 8

$n(S) = 8$

a) Atleast one head

The probability of getting atleast one head is

A = {(HHH), (HHT, (HTH), (THH), (TTH), (THT), (HTT)}

$$P(A) = \frac{n(A)}{n(S)}$$

$$= 7/8$$

b) Exactly two head

$B = \{THH, HHT, HTH\}$

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$$

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4) Four coins are tossed. find the probability of getting 2 heads & 2 tail.

The possible outcomes are

$S = \{(HHHH), (HHHT), (HTHT), (THHH), (THTH),$
$(TTHH), (HHTT), (TTTT), (TTTH), (HTTT), (HTHH)$
$(HTTH), (THHT), (HHTH), (THTT), (TTHT)\}$

$n(S) = 16$

The probability of getting two heads and two tail

$A = \{(HTHT), (THTH), (TTHH), (HHTT),$
$(HTTH), (THHT)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{16} = \frac{3}{8}$$

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5. A perfect die is tossed twice.. Find the the probability of getting a total of 9.

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(S) = 36$

$A = \{(3,6)(4,5)(5,4)(6,3)\}$

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

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6. Two dice are throwned. find the probability

a) Total number on the dices 8

b) The first dies shows 6

c) The total number on the die is greater than 8. 5/18

d) The total number on the die is 13 0

e) Both the die shows the same 1/6

f) Sum of the number shown by the dies less than 5 1/6

g) Sum of the number shown by the dies exactly 6 5/36

$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$
$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$
$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$
$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$
$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$
$(6,1)(6,2)(6,3)(6,4)(6,5),(6,6)\}$

$n(S) = 36$

a) $A = \{(2,6)(3,5)(4,4)(5,3)(6,2)\}$

$n(A) = 5$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

b) ~~The~~ probability of getting first dies shows 6

$\cancel{A}=\{(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

$n(\cancel{A})=6$

$p(A)=\frac{n(A)}{n(S)}=\frac{\cancel{6}}{3\cancel{6}}=\frac{1}{6}$

c) The probability of getting total number of die is greater than 8

$A=\{(3,6),(4,5)(4,6)(5,4)(5,5)(5,6),(6,3)$
$(6,4)(6,5)(6,6)\}$

$n(A)=10$

$p(A)=\frac{n(A)}{n(S)}=\frac{\cancel{10}^{5}}{\cancel{36}_{18}}=\frac{5}{18}$

d) The probability of getting total number of the die is 13

$n(A)=0$

$p(A)=\frac{n(A)}{n(S)}=\frac{0}{36}=0$

e) The probability of getting both the die shows the same

$A=\{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$

$n(A)=6$

$p(A)=\frac{n(A)}{n(S)}=\frac{\cancel{6}^{1}}{\cancel{36}_{6}}=1/6$

f) The probability of getting sum of the number shown by the dies less than 5

$$A = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2), (3,1)\}$$

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

g) The probability of getting sum of the number shown by the dies exactly by 6

$$A = \{(1,5)\ (2,4)\ (3,3)\ (4,2)\ (5,1)\}$$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

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7. Tickets are numbered from 1 to 100. Their are well shuffled & ticket is choosen at random. What is the probability that the drawn ticket

i) an even number
ii) A number 5 or a multiple of 5
iii) A number which is greater than 75
iv) A number which is a square

Sol The total number of tickets

$$S = \{1, 2, 3 \dots 100\}$$

$$n(S) = 100.$$

i) The probability of getting an even number.

$$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100\}$$

$n(A) = 50$

$$P(A) = \frac{n(A)}{n(S)} = \frac{50}{100} = \frac{1}{2}$$

ii) The probability of getting a number 5 or multiple of 5

$B = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100\}$

$n(B) = 20$

$$P(B) = \frac{n(B)}{n(S)} = \frac{20}{100} = \frac{1}{5}$$

iii) The probability of getting a number greater than 75

$C = \{76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100\}$

$n(C) = 25$

$$P(C) = \frac{n(C)}{n(S)} = \frac{25}{100} = \frac{1}{4}$$

iv) The probability of getting a number which is square

$D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$n(D) = 10$

$$P(D) = \frac{n(D)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

8) If a pair of dies is throwned find the probability that the sum is neither 7 or 11

two dies

$$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$$
$$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$$

$n(S) = 36$

The probability of getting sum is neither 7 or 11

$$A = \{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (5,6)$$
$$(6,1)\ (6,5)\}$$

$n(A) = 8$

$n(S) - n(A)$

Neither 7 or 11 $\Rightarrow 36 - 8$

$n(A) = 28$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{36} = \frac{7}{9}$$

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9) A bag contains 4 white ball, 6 black balls. Two balls are drawn at random. What is the probability that i) Both are white?

ii) Both are black

iii) one white and one black ball

C → Complement.

No. of white ball = 4

No. of. black ball = 6

Total no. of balls is $\overline{10}$ (x.)

$n(S) = {}^{10}C_2 \rightarrow$ use calculate -> press. 10 -> Then (Shift) + ($\div$) it shows -> Then press 2 It shows = 45

$n(S) = 45$

i) Both are white $= {}^4C_2$

$n(A) = 6$

$p(A) = \frac{n(A)}{n(S)} = \frac{6}{45}$

ii) Both are black $= {}^6C_2$

$n(B) = 15$

$P(B) = \frac{n(B)}{n(S)} = \frac{15}{45}$

iii) one white & one black ball

${}^4C_1 \times {}^6C_1$

$n(C) = 24$

$P(C) = \frac{n(C)}{n(S)} = \frac{24}{45}$

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10. Their are 2 red, 3 green and 4 black ball of identical size in an urn 3 balls are drawn at random find the probability of getting ${}^9C_3$
 i) Their are of different colours ${}^2C_1 \times {}^3C_1 \times {}^4C_1$
 ii) 2 are green & 1 is black ${}^3C_2$ ${}^4C_1$
 iii) 2 are red ${}^2C_2$
 iv) Atleast 1 is black

${}^4C_1 \times {}^4C_2 \times {}^4C_3 \times {}^4C_4$

No.of. red balls = 2
No.of green balls = 3
No.of. black balls = 4

Total no.of balls = 9

$n(S) = 9C_3$

$n(S) = 84$

i) Their are of different colours

$2C_1 \times 3C_1 \times 4C_1$

$2 \times 3 \times 4 = 24$

$n(A) = 24$

$p(A) = \frac{24}{84}$

ii) 2 are green and 1 is back

$3C_2 \times 4C_1 = 3 \times 4 = 12$

$n(B) = 12$

$p(B) = \frac{n(B)}{n(S)} = \frac{12}{84}$

iii) 2 are red

$2C_2 = 1$

$n(C) = 1$

$p(C) = \frac{n(C)}{n(S)} = \frac{1}{84}$

iv) Atleast 1 is black

$\cancel{2}4C_1 \times 4C_2 \times 4C_3 \times 4C_4 = 4 \times 6 \times 4 \times 1 = 96$

$n(D) = 96$

$p(D) = \frac{n(D)}{n(S)} = \frac{96}{84}$

11 In a play of 2 dies the thrower loses if it is 1st throw 2, 4, or 12. He wins if his 1st throw is 5 or 11. Find the ratio between his probability of losing and probability of winning in the 1st throw.

Two dices are thrown = 36

$n(S) = 36$

i) If it is the 1st throw 2, 4 or 12 is

$A = \{(1,1)(1,3)(2,2)(3,1)(6,6)\}$

$n(A) = 5$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

ii) If his 1st throw is 5 or 11

$B = \{(1,4)(2,3)(3,2)(4,1)(5,6)(6,5)\}$

$n(B) = 6$

$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

iii) The ratio between his probability of losing to that winning in 1st throw

$C = \frac{\frac{5}{36}}{\frac{1}{6}}$

$= \frac{5}{36} \times \frac{6}{1}$

$= \frac{5}{6}$

$= 5:6$

12) A Sub Commitee of 6 member is to be formed out of a group consisting of 7 men and 4 women Calculate the probability that the Sub Committee will consist of

i) Exactly 2 womens
ii) Atleast 2 womens.

Sol

Total number of men = 7

Total number of women = 4

The total number men & women $= \overline{\underline{11}}$

A sub commitee member is

$n(S) = 11C_6 = 462$ $\boxed{11C_6}$

i) Exactly 2 womens:-

$= 7C_4 \times 4C_2 = 35 \times 6$

$n(A) = 210$

$P(A) = \frac{n(A)}{n(S)} = \frac{210}{462} = \frac{5}{11}$

ii) Atleast 2 womens:

$7C_4 \times 4C_2 + 7C_3 \times 4C_3 + 7C_2 \times 4C_4$

$n(B) = 371$

$P(B) = \frac{n(B)}{n(S)} = \frac{371}{462} = \frac{53}{66}$

13) One Card is drawn at random from a well shuffled pack of 52 Cards. What is the probability that it will be a) a diamond b) a queen.

Total number of Card is 52

$n(S) = 52$

Spade
Heart
Club
Diamond
] 13

a) There are 13 diamond card in a pack of 52 cards

$n(A) = 13$

$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = 1/4$

King
Queen
Jack
Ace
] 4

b) There are 4 queens card in a pack of 52 Cards.

$n(B) = 4$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

14) Two Cards are drawn from a pack of Cards at random. What is the probability that it will be $52C_2 = 1326$

a) a diamond and a heart $13C_1 \times 13C_1 = 169$

b) a King and a queen $4C_1 \times 4C_1 = 16$

c) two kings. $4C_2 = 6$

Total number of card is

$$^{52}C_2 = 1326$$

$$n(S) = 1326$$

a) The probability of getting a diamond and a heart card

$$^{13}C_1 \times {^{13}C_1} = 169$$

$$n(A) = 169$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{169}{1326}$$

b) The probability of getting a king and a queen

$$^{4}C_1 \times {^{4}C_1} = 16$$

$$n(B) = 16$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{16}{1326}$$

c) The probability of getting a two kings

$$^{4}C_2 = 6$$

$$n(C) = 6$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{1326}$$

15. A Candidate is Selected for interview in 3 different post. Their are 3 Candidates for the 1st post, 4 for the 2nd post and 2 for the 3rd post. What is a probability that he will be Selected for one of the post?

Number of Candidate of 1st post = 3

Number of Candidates in 2nd post = 4

Number of Candidates in 3rd post = 2

The probability that the candidate is Selected for 3 posts are $1/3, 1/4, 1/2$ respectively

The probability that he is not Selected for 3 posts are $2/3, 3/4, \frac{1}{2}$

The probability that he will be selected for one of the post.

= 1 − probability that he will not be selected for any of the post

$$= 1 - \left[\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2}\right]$$

$$= 1 - \left(\frac{6}{12} \times \frac{1}{2}\right)$$

$$= 1 - \frac{3}{12}$$

$$= \frac{9}{12} = 3/4$$

16. Two urns contain respectively 10 white, 6 red & 9 black balls and 3 white, 7 red and 15 black balls. One ball is drawn from each urn, find the probability that i) both balls are red, ii) both balls are of the same colour

Sol

Urns	White	Red	Black	Total
Urn I	10	6	9	25
Urn II	3	7	15	25

i) The probability that both balls are red

$$\frac{6C_1}{25C_1} \times \frac{7C_1}{25C_1}$$

$$= \frac{42}{625}$$

ii) The probability that both balls are of the same colour.

$$\frac{10C_1}{25C_1} \times \frac{3C_1}{25C_1} + \frac{6C_1}{25C_1} \times \frac{7C_1}{25C_1} + \frac{9C_1}{25C_1} \times \frac{15C_1}{25C_1}$$

$$= \left(\frac{10}{25} \times \frac{3}{25}\right) + \left(\frac{6}{25} \times \frac{7}{25}\right) + \left(\frac{9}{25} \times \frac{15}{25}\right)$$

$$= \frac{30}{625} + \frac{42}{625} + \frac{135}{625}$$

$$= \frac{207}{625}$$

17 What is the chance of getting a king in the draw from the pack of 52 cards.

The total number of cards in pack = 52

$n(S) = 52$

Total number of kings in pack = 4

$n(A) = 4$

The probability of drawing a king

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{52}$$

$$P(A) = \frac{1}{13}$$

18. The probabilities of 3 students A, B and C solving a problem in statistics are $\frac{1}{2}, \frac{1}{3}$ & $1/4$. A problem is given to all the three student what is the probability that.

i) No one will be solve the problem.

ii) Only one will solve the problem.

iii) Atleast one will solve the problem.

Sol Probability of A solving the problem = $1/2$

Probability of B solving the problem = $1/3$

Probability of C solving the problem = $1/4$

Probability of A not solving the problem is $1 - \frac{1}{2} = \frac{1}{2}$

Probability of B not solving the problem is $1 - 1/3 = 2/3$

probability of c not solving the problem is

$1-\frac{1}{4}=3/4$

a) Non one will be solve the problem.

$$A=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$$

$$n(A)=\frac{1}{4}$$

b) probability of only one will solve the problem.

$$=\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times 1/4\right)$$

$$=\frac{1}{4}+\frac{1}{8}+\frac{1}{12}$$

(?) $=\frac{11}{24}$

c) probability of atleast one will solve the problem.

= 1- probability of no one will solve the problem

$$=1-\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right)$$

$$=1-\frac{1}{4}$$

$$=3/4$$

Baye's Theorem :-

10M

Statement :

If $E_1, E_2 \dots E_n$ are mutually disjoints event with $P(E_i) \neq 0$ where $i = 1, 2, \dots n$ then for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$ then

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

proof :

Given : $A \subset \bigcup_{i=1}^{n} E_i$ to prove

$$TP : P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

We have

$$A = A \cap \left(\bigcup_{i=1}^{n} E_i\right)$$

$$= A \cap (E_1 \cup E_2 \cup \dots \cup E_n)$$

$$= (A \cap E_1) \cup (A \cap E_2) \cup \dots \cup (A \cap E_n)$$

$$A = \bigcup_{i=1}^{n} (A \cap E_i)$$

Taking probability on both sides

$$P(A) = P\left[\bigcup_{i=1}^{n} (A \cap E_i)\right] \rightarrow (1)$$

By multiplication theorem for dependent event :

$P(A \cap E_i) = P(E_i)\, P(A|E_i)$ if $P(E_i) > 0 \rightarrow$ (2)

$P(A \cap E_i) = P(A) . P(E_i/A)$ if $P(A) > 0 \rightarrow$ (3)

from (3)

$$P(E_i/A) = \frac{P(A \cap E_i)}{P(A)} \rightarrow (4)$$

Sub (2) in (1)

$$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A/E_i) \rightarrow (5)$$

Sub (2) & (5) in equ (4)

$$P(E_i/A) = \frac{P(E_i)\, P(A|E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A|E_i)}$$

Standard Deviation

Bernoulli Trial:

Each trial has two possible outcome generally called success and failure such trial is known as bernoulli trial.

Binomial Experiment:

An experiment consisting of repeated number of bernoulli trial is called binomial experiment.

Binomial random Variable:

Let s be the number of success in n repeated independent bernoulli trial with probability p of success for each trial.

Then s is called the binomial random variable with parameter p and n

Binomial distribution:

Let s be the binomial random variable with parameter n and p then binomial distribution is defined by

$$P(X=x) = nC_x \, p^x q^{n-x}, \quad X = 0, 1, 2, \ldots n$$

$$p+q=1$$

$$p=1-q$$

$$q=1-p$$

Properties of binomial distribution:

i) Binomial distribution as two parameter n and p (or q)

ii) Mean (Average) $E(X) = np$

iii) Variance (x) or $Var(x) = npq$

Standard deviation (S.D)

$$SD = \sqrt{npq}$$

Skewness $\beta_1^2 = \frac{(q-p)^2}{npq}$

Kurtosis $\beta_2 = 3 + \frac{1-6pq}{npq}$

Binomial distribution is symmetrical if $p = q = 0.5$

It is a possitively ~~Kurto~~ Skewed if $p < 0.5$ and it is negatively skewed $p > 0.5$

1. For binomial distribution with parameters $n = 5$, $p = 0.3$. Find the probability of getting

   i) Atleast 3 Success 0.16308 but x

   ii) Atmost 3 Success 0.9692 $3 + (\text{shift} \div) C$

   iii) Exactly 3 failure

i) Atleast 3 Success $n = 5, p = 0.3$ $q = 1 - 0.3 = 0.7$

$p(X=3) + p(X=4) + p(X=5)$

$= nC_x p^x q^{n-x} + nC_x p^x q^{n-x} + nC_x p^x q^{n-x}$

$= 5C_3 (0.3)^3 (0.7)^{5-3} + 5C_4 (0.3)^4 (0.7)^{5-4} + 5C_5 (0.3)^5 (0.7)^{5-5}$

$= 10(0.027)(0.49) + 5(0.0081)(0.7) + 1(0.00243)(1)$

$= 0.1323 + 0.02835 + (0.00243)$

$= 0.16308$

ii) Atmost 3 Success.

$P(x \le 3)$ $1 - P(x > 3)$

$= P(x=0) + p(x=1) + p(x=2) + p(x=3)$

$= 5C_0(0.3)^0(0.7)^5 + 5C_1(0.3)^1(0.7)^4 + 5C_2(0.3)^2(0.7)^3 + 5C_3(0.3)^3(0.7)^2$

$= 1(1)(0.16807) + 5(0.3)(0.2401) + 10(0.09)(0.343) + 10(0.027)(0.49)$

$= 0.16807 + 0.36015 + 0.3087 + 0.1323$

$= 0.96922$

iii) Exactly 3 failure.

$p(x=3) = 5C_3\,(0.3)^3(0.7)^{5-3}$

$= 5C_3\,(0.3)^3(0.7)^2$

$= 10(0.027)(0.49)$

$= 0.1323.$

Probability distribution Function (pdf).

Probability distribution functions can be classified into two categories.

- Discrete probability distribution
- Continuous probability distribution

Discrete probability distribution: Assumes that the outcomes of a random variable under study can take on only integer values, such as:

- A book shop has only 0, 1, 2.. copies of a particular title of a book.
- A consumer can buy 0, 1, 2, ... shirts, pants, etc...

If the random variable x is discrete, its probability distribution called probability mass function (pmf) must satisfy following two conditions.

i) The probability of a any specific outcome for a discrete random variable must be between 0 & 1. Stated mathematically, $0 \leq f(x=K) \leq 1$, for all value of K

ii) The sum of the probabilities over all possible values of a discrete random variable must equal 1. Stated mathematically, $\sum_{\text{all } k} f(x=k) = 1$.

A continuous probability distribution assumes that the outcomes of a random variable can take on only value in an interval such as:

- product costs & prices
- Floor area of a house, office etc.

If the random variable x is continuous, then its probability density function must satisfy following two conditions.

i) $p(x) \geq 0 : -\infty < x < \infty$ (non-negativity condition)

ii) $\int_{-\infty}^{\infty} p(x)\,dx = 1$ (Area under the continuous curve must total 1)

Meaning of Discrete probability distribution:

A probability distribution in which the random variable is permitted to take on only integer values.

Continuous probability distribution:

A probability distribution in which the random variable is permitted to take any value within a given range.

Continuous probability distribution :

This functions are used to find probabilities associated with random variable values $x_1, x_2 \ldots x_n$ in a given interval or range, say (a,b). In other words, these probability are determined by finding the area under the pdf between the values a and b. Mathematically, the area under pdf between a and b is given by

$$f(a \le x \le b) = f(b) - f(a) = \int_a^b f(x)\,dx$$

We can express $f(a \le x \le b)$ in terms of a distribution function, $f(x)$, provided it is differentiable. That is,

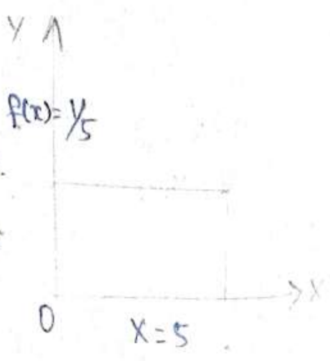
$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left\{\int_a^b f(x)\,dx\right\}$$

Ill - Consider the function, $f(x) = \begin{cases} a & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$

For $f(x)$ to be a pdf, the condition,

$\int_{-\infty}^{\infty} f(x)\,dx = 1$ must be satisfied; which is true

$$\int_0^5 a\,dx = 1, \text{ i.e } a = \frac{1}{5}$$



Since $a > 0$, the function, $f(x) \ge 0$. Thus $f(x)$ satisfies both the conditions for a pdf.

1) A brokerage survey report that 30% of individual investors have used a discount broker that is, which does not charge the full commission. In a random sample of 9 individuals, what is the probability that

i) Exactly 2 of the sampled individual have used a discount broker.

ii) Not more than 3 have used a discount broker.

iii) Atleast 3 of them have used a discount broker.

$n = 9$, $p = 30\% = 0.3$, $q = 1-p = 1-0.3 = 0.7$

i) Exactly 2

$$P(X=x) = nC_x \, p^x q^{n-x}$$

$$P(X=2) = 9C_2 \, (0.3)^2 (0.7)^{9-2}$$

$$= 36\,(0.09)\,(0.0823543)$$

$$= 0.26683.$$

ii) Not more than 3

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= nC_x p^x q^{n-x} + nC_x p^x q^{n-x} + nC_x p^x q^{n-x} + nC_x p^x q^{n-x}$$

$$= 9C_0 (0.3)^0 (0.7)^{9-0} + 9C_1 (0.3)^1 (0.7)^8 + 9C_2 (0.3)^2 (0.7)^7 + 9C_3 (0.3)^3 (0.7)^6$$

$$= (1)(1)(0.040354) + 9\,(0.3)(0.05764801) + 36(0.09)(0.0823543) + 84\,(0.027)\,(0.117649)$$

$$= 0.040353607 + 0.155649627 + 0.266827932 + 0.266827932$$

$$= 0.729659098 \; (0.73)$$

iii) Atleast 3

$$P(X \geq 3) = 1 - P(X<3)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - [nC_x p^x q^{n-x} + nC_x p^x q^{n-x} + nC_x p^x q^{n-x}]$$

$$= 1 - [9C_0 (0.3)^0 (0.7)^{9-0} + 9C_1 (0.3)^1 (0.7)^{9-1} + 9C_2 (0.3)^2 (0.7)^{9-2}]$$

$$= 1 - [0.04035 + 0.155655 + 0.26683]$$

$$= 1 - 0.462835$$

$$= 0.5372$$

2. Mr. Gupta applys for a personal loan of Rs. 1,50,000 from a nationalized bank to repair his house. The loan offer informed him that over the years the bank has received about 2920 loan application per year and the probability of approval was on average above 0.85

i) Mr Gupta wants to know the average and standard deviation of the number of loans approved per year

ii) Suppose bank actually received 2654 loan application per year with an approval probability of 0.82 what are the mean and SD now.

i) $n = 2920$ $p = 0.85$ $q = 1-p = 1-0.85$
$= 0.15$

Mean (or) average $= np$
$= 2920 \times 0.85$
$= 2482$

$S.D = \sqrt{npq}$
$= \sqrt{2920 \times 0.85 \times 0.15}$
$= \sqrt{372.3}$
$= 19.295$

ii) $n = 2654$ $p = 0.82$ $q = 1-0.82$
$= 0.18$

Mean $= np$
$= 2654 \times 0.82$
$= 2176.28$

$S.D = \sqrt{npq}$
$= \sqrt{2654 \times 0.82 \times 0.18}$
$= 19.792$

3. Suppose 10% of new scooters will require warranty service within the 1st month of its sale. A scooter manufacturing company sells 1000 scooters in a month.

i) Find the mean, SD of scooters that require warranty service

ii) Calculate the movement coefficient of skewness and kurtosis of the distribution.

i) $n = 1000$ $\quad p = 10\%$ $\quad q = 1 - p$

$= 0.1$ $\quad = 1 - 0.1$

$= 0.9$

$\text{mean} = np$

$= 1000 \times 0.1$

$= 100$

$S.D = \sqrt{npq}$

$= \sqrt{1000 \times 0.1 \times 0.9}$

$= \sqrt{90}$

$= 9.486$ (Approximately)

$= 10$

$\beta_1^2 =$

ii) Skewness $= \dfrac{(q-p)^2}{npq}$ (or) $\dfrac{q-p}{\sqrt{npq}}$

β_1^2

$= \dfrac{(0.9 - 0.1)^2}{90}$ $\quad \beta_1 = \dfrac{0.8}{9.486}$

$= \dfrac{(0.8)^2}{90} = \dfrac{0.64}{90}$ $\quad = 0.084$

$\beta_1^2 = 0.00711$

$\beta_1 = \sqrt{0.00711}$

$= 0.084$

Kurtosis $= 3 + \dfrac{1 - 6pq}{npq}$

$= 3 + \left(\dfrac{1 - 6(0.09)}{90} \right)$

$= 3 + \left(\dfrac{0.46}{90} \right)$

$= 3 + 0.00511$

$= 3.00511$

The incidence of occupational disease in a industry is such that the workers have 20 percent chance of suffering from it. What is the probability that out of six workers 4 or more will come in contact of the disease?

$$n=6 \quad p=20/100 = 0.2 \quad q=1-p = 1-0.2 = 0.8$$

$$P(X \geq 4) = P(x=4)+P(x=5)+P(x=6)$$

$$= nC_x p^x q^{n-x} + nC_x p^x q^{n-x} + nC_x p^x q^{n-x}$$

$$= 6C_4 (0.2)^4 (0.8)^{6-4} + 6C_5 (0.2)^5 (0.8)^{6-5} + 6C_6 (0.2)^6 (0.8)^{6-6}$$

$$= 15(0.016)(0.64) + 6\,[0.00032)(0.8) + 1(0.000064)(1)$$

$$= 0.01695$$

2. A multiple choice test contain 8 questions with 3 answer to each question of which only one is correct. A student answer each question by rolling a balanced dice and checking the first answer if he gets 1 or 2, the second answer if he get 3 or 4, and the third answer if he gots 5 or 6. To get a distinction the student must secure at least 75 percent correct answers. If there is no negative marking. what is the probability that the student secures a distinction?

Sol

$p = 1/3 = 0.33 \qquad q = 1-p$
$= 1-0.33$
$= 0.67$

$P(X \geq 6) = p(X=6) + p(x=7) + p(x=8)$

$= 8C_6 (0.33)^6 (0.67)^{8-6} + 8C_7 (0.33)^7 (0.67)^{8-7} +$
$8C_8 (0.33)^8 (0.67)^{8-8}$

$= [28(0.0012914 67969)(0.4489)] + [8(0.000461844298)(0.67)] + [1(0.0001464 08618)(1)]$

$= 0.0162327192 + 0.0024754 8543 7 + 0.0001464 08618$

$= 0.01885461326 \text{ (or) } 0.0196.$

poisson distribution

Poisson probability distribution is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \; x = 0, 1, 2, \ldots$$

lamida

factorial

↳ Shift + x^{-1} = !

Where $\lambda = np$

Note: (properties of poisson distribution)

mean $= \lambda$

Variance $= \lambda$

Standard deviation $= \sqrt{\lambda}$

Skewness $\beta_1 = \frac{1}{\lambda}$

Kurtosis $\Rightarrow \beta_2 = 3 + \frac{1}{\lambda}$

Addictive property of binomial of random variable:

⇒ If X_1 and X_2 are two independent binomial random variable with parameters (p, n_1) (p, n_2) then $(X_1 + X_2)$ is a binomial random variable with parameter $(pn_1 + pn_2)$

Addictive property of poisson distribution:

⇒ If X_1 and X_2 are two independent poisson random variable with parameters λ_1 and λ_2 then $X_1 + X_2$ is a poisson random variable with a parameter $\lambda_1 + \lambda_2$

poisson frequency distribution:

⇒ Let a poisson experimental consist of n independent [illegible] trial.

Let the experiment under similar condition be repeated N times. Then poisson frequency distribution is $P(X=x) = \frac{N e^{-\lambda} \lambda^{x}}{x!}$

Ex - * Number of defective item produced in a factory.

* Number of death due to rare diseases

* Number of mistakes committed by a typist

1. If X is a poisson random variable such that $P(X=1) = 0.3$, $P(X=2) = 0.2$ find $P(X)=0$

<u>Sol</u>

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(X=1) = 0.3$$

$$\frac{e^{-\lambda}\lambda^{1}}{1!} = 0.3$$

$$e^{-\lambda}\lambda = 0.3 \rightarrow (1)$$

$$P(X=2) = 0.2$$

$$\frac{e^{-\lambda}\lambda^{2}}{2!} = 0.2$$

$$\frac{e^{-\lambda}\lambda^{2}}{2} = 0.2 \rightarrow (2)$$

$$(1) \div (2)$$

$$\frac{e^{-\lambda}\lambda}{\frac{e^{-\lambda}\lambda^{2}}{2}} = \frac{0.3}{0.2}$$

$$\frac{2e^{-\lambda}\lambda}{e^{-\lambda}\lambda^2} = 1.5$$

$$\frac{2}{\lambda} = 1.5$$

$$2 = 1.5\lambda$$

$$\frac{2}{1.5} = \lambda$$

$$\lambda = 1.33$$

$$P(x=0) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \frac{e^{-1.33}(1.33)^0}{0!}$$

$$= \frac{e^{-1.33}(1)}{1} \qquad e = \text{shift} + \text{in}$$

$$= e^{-1.33} \quad \text{shift in } (-1.33)$$

$$= 0.2644$$

2. Find the probability atmost five (5) defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective. Give that $e^{-4} = 0.0183$

$$\lambda = np$$

$$n = 200 \quad p = 2\% = \frac{2}{100} = 0.02$$

$$\lambda = np$$

$$= 200 \times 0.02$$

$$\lambda = 4$$

Atmost 5 defective

$$P(X\le 5) = p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)+p(x=5)$$

$$= \frac{e^{-4}\lambda^0}{0!}+\frac{e^{-4}4^1}{1!}+\frac{e^{-4}4^2}{2!}+\frac{e^{-4}4^3}{3!}+\frac{e^{-4}4^4}{4!}+\frac{e^{-4}4^5}{5!}$$

$$= 0.0183+0.0183(4)+\frac{0.0183(\cancel{16}\ 8)}{\cancel{2}}+\frac{0.0183(64)}{6}+\frac{0.0183(256)}{24}+\frac{0.0183(1024)}{120}$$

$$= 0.0183+0.0732+0.1464+0.1952+0.1952+0.1561$$

$$= 0.7844.$$

3. If 3% of electric bulbs manufactured by a company are defective. find the probability that in a sample of 100 bulb. Exactly 5 bulbs are defective.

$$n = 100 \quad p = 3\% = 0.03$$

$$\lambda = np = 100\times 0.03 = 3$$

$$P(X=5) = \frac{e^{-3}(3)^5}{5!}$$

$$= \frac{e^{-3}(243)}{120}$$

$$= \frac{0.049787(243)}{120}$$

$$= \frac{12.098}{120}$$

$$= 0.1008$$

4. A Manufacture of pins knows that two (2) % of this products are defective. If he says (selling) pins in boxs of 100 and ~~gave~~ guanteen that more than 4 pins will be defective. What is the probability to meet the gurantee quality

$$n = 100 \quad p = 0.02$$

$$\lambda = np$$
$$= 100 \times 0.02$$
$$= 2.$$

$$p(x \geq 4) = 1 - p(x \leq 4)$$

$$= 1 - [p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)]$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^x}{x!} + \frac{e^{-\lambda}\lambda^x}{x!} + \frac{e^{-\lambda}\lambda^x}{x!} + \frac{e^{-\lambda}\lambda^x}{x!} + \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$$= 1 - \left[\frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)^1}{1!} + \frac{e^{-2}(2)^2}{2!} + \frac{e^{-2}(2)^3}{3!} + \frac{e^{-2}(2)^4}{4!}\right]$$

$$= 1 - \left[\frac{0.13533(1)}{1} + \frac{0.13533(2)}{1} + \frac{0.13533(4)}{2} + \frac{0.13533(8)}{6} + \frac{0.13533(16)}{24}\right]$$

$$= 1 - [0.13533 + 0.27066 + 0.27066 + 0.18044 + 0.09022]$$

$$= 1 - [0.94731]$$

$$= 0.05269$$

5. An insurance Company as discovered that only about 0.1% of the population is involved in a certain type of accident each year. If its 10,000 policy holder were randomly selected from the population. What is the probability that not more than 5 of its claim are involved in such an accident next year.?

$n = 10{,}000 \quad p = 0.003 \quad \lambda = np = 10{,}000 \times 0.0$
$= 10$

$P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= \frac{e^{-10}(10)^0}{0!} + \frac{e^{-10}(10)^1}{1!} + \frac{e^{-10}(10)^2}{2!} + \frac{e^{-10}(10)^3}{3!} + \frac{e^{-10}(10)^4}{4!} + \frac{e^{-10}(10)^5}{5!}$

$= \frac{0.000045(1)}{1} + \frac{0.000045(10)}{1} + \frac{0.000045(100)}{2} + \frac{0.000045(1000)}{6} + \frac{0.000045(10000)}{24} + \frac{0.000045(100000)}{120}$

$= 0.000045 + 0.00045 + 0.00225 + 0.0075 + 0.01875 + 0.0375$

$= 0.066$

6. A random Variable X follows a poisson distribution Such that $p(X=2) = p(X=1)$ find $p(X=0)$

$$P(X=2) = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$p(X=1) = \frac{e^{-\lambda}\lambda^1}{1!}$$

Given

$$p(X=2) = p(X=1)$$

$$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-\lambda}\lambda^1}{1!}$$

$$\frac{\lambda}{2} = 1$$

$$\lambda = 2$$

$$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!}$$

$$= \frac{e^{-2}(2)^0}{0!}$$

$$= 0.1353$$

7. At a busy traffic junction a probability of an individual have an accident $p=0.001$. However during a certain part of the day 1000 cars pass through the junction. What is a probability that 2 (or) more accident occur during that period. $(e^{-0.1} = 0.9048)$

$$P = 0.001$$

$$n = 1000$$

$$\lambda = np$$

$$= 1000 \times 0.001$$

$$\lambda = 1$$

Probability that 2 or more accident occurs during that period $p(x \geq 2)$

$= 1 - p(x<2)$

$= 1 - [p(x=0) + p(x=1) + p(x=2)]$

$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!} + \frac{e^{-1}(1)^2}{2!}\right]$

$= 1 - [0.36787$

$= 1 - 0.7356$

8) A factory employee a large number of workers find that over a period of time the average absentees range is 3 workers per shift. Calculate the probability that in a given shift.

1) Exactely 2 workers will be absent.
2) More than 4 workes will be absent.

$\lambda = 3$

i) Exactely 2 workers

$$P(x=2) = \frac{e^{-3}(3)^2}{2!}$$

$$= \frac{0.04979 \times 9}{2}$$

$$= 0.224$$

ii) More than 4 workers will be absent.

$$P(x \geq 4) = 1 - P(x \leq 4)$$

$$= 1 - [p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)]$$

$$= 1 - \left[\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!} + \frac{e^{-3}(3)^4}{4!}\right]$$

$$= 1 - \left[\frac{0.04979(1)}{1} + \frac{0.04979(3)}{1} + \frac{0.04979(9)}{2}\right.$$

$$+\frac{0.04979(27)}{6}+\frac{0.04979(81)}{24}$$

$$=1-[0.04979+0.14937+0.224055+0.224055+0.16804125]$$

$$=1-0.8153$$

$$=0.1846$$

9. After correcting the proof of the first 50 pages of a book, it is found that on the average there are 3 error per 5 pages Use poisson probability and estimate the number of pages with 0, 1, 2, 3 errors in whole books are 1000 pages ($e^{-0.6}=0.5488$)

$$\lambda=3/5=0.6$$

i) No. of pages containing 0 error

$$NP(X=0)=1000\left(\frac{e^{-0.6}(0.6)^0}{0!}\right)$$

$$=1000\left(\frac{0.5488\times1}{1}\right)$$

$$=548.8$$

ii) No. of pages containing 1 error

$$NP(X=1)=1000\left(\frac{e^{-0.6}(0.6)^1}{1!}\right)$$

$$=1000\left(\frac{0.5488\times0.6}{1}\right)$$

$$=329.28$$

iii) No. of. pages containing 2 error

$$Np(x=2) = 1000\left(\frac{e^{-0.6}(0.6)^2}{2!}\right)$$

$$= 1000\left(\frac{0.5488 \times 0.36}{2}\right)$$

$$= 1000\left(\frac{0.1975}{2}\right)$$

$$= 1000\,(0.09875)$$

$$= 98.75$$

iv) No. of. pages containing 3 error

$$N\,p(x=3) = 1000 \times \frac{e^{-0.6}(0.6)^3}{3!}$$

$$= 1000 \times \frac{0.5488 \times 0.216}{6}$$

$$= 1000 \times \frac{0.1185}{6}$$

$$= 1000 \times 0.01975$$

$$= 19.75$$

10) One fifth percent of the blade produce by a blade manufacturing factory turn out to be defective. The blades are supplied in package of 10. Use poisson distribution to calculate the approximate number of package containing no defective, 1 defective, 2 defective blades respectively. In a consignment of 1 lakh package $[e^{-0.02} = 0.9802]$

$P = 1/5\%$

$= 0.2\% = 0.002$

$n = 10 \quad \lambda = np = 10 \times 0.002$

$= 0.02$

i) No. of package Containing no defective

$$Np(X=x) = N\left(\frac{e^{-\lambda}\lambda^{x}}{x!}\right)$$

$N = 1,00,000$

$$Np(X=0) = 100000\left(\frac{e^{-0.02}(0.02)^0}{0!}\right)$$

$= 100000 \times 0.9802 \times 1$

$= 98020.$

ii) No. of packages Containing one defective

$$Np(X=1) = N\left(\frac{e^{-\lambda}\lambda^{x}}{x!}\right)$$

$$= 1,00,000\left(\frac{e^{-0.02}(0.02)^1}{1!}\right)$$

$= 1,00,000 \times 0.9802 \times 0.02$

$= 1960.4$

iii) No. of. packages Containing two defective

$$Np(X=2) = 1,00,000\left(\frac{e^{-0.02}(0.02)^2}{2!}\right)$$

$$= 1,00,000\left(\frac{0.9802 \times 0.0004}{2}\right)$$

$= 1,00,000\,(0.0001\,9604)$

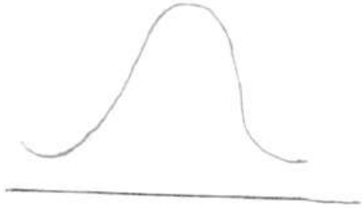
$= 19.604$

Normal Distribution:

The approximation of binomial when n is large and p is not closed to 0 (or) 1 is called normal distribution.

Characteristics of normal distribution:

A diagram of a normal distribution given below is called normal curve.



i) The normal distribution is a symmetrical distribution and graph of the normal distribution is bell shaped.

ii) The curve has a single peak point (i.e the distribution is uni model)

iii) The mean of the normal distribution lies at the centre of the normal curve.

iv) Because of the symmetric of the normal curve, the median and mode are also at the centre of the normal curve.

v) The parameter mean and standard deviation (μ, σ) completely determined the distribution.

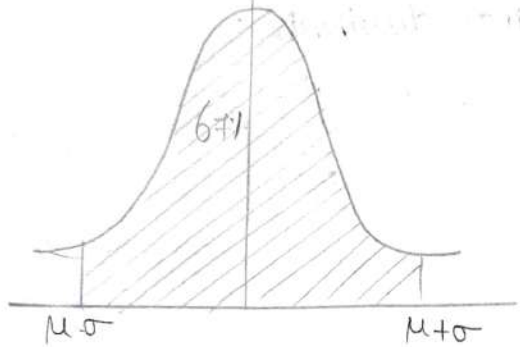
vi) Area property:

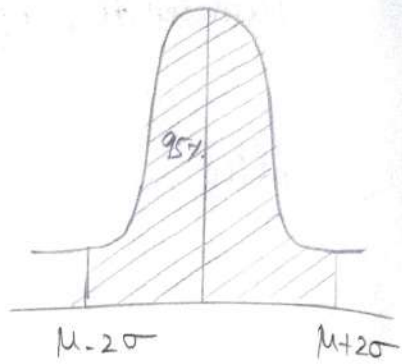
In a normal distribution

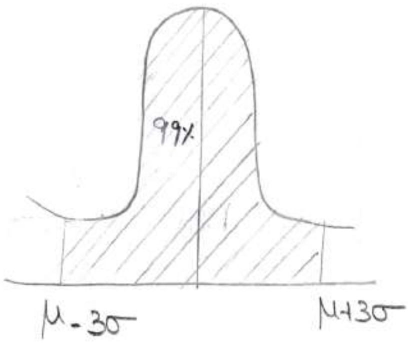
i) Above 67% of the observation will lie between mean $\pm$ standard deviation $(\mu \pm \sigma)$

ii) Above 95% of the observation will lie between mean ± 2 standard deviation ($\mu \pm 2\sigma$)

iii) Above 99% of the observation will lie between mean ± 3 standard deviation ($\mu \pm 3\sigma$)

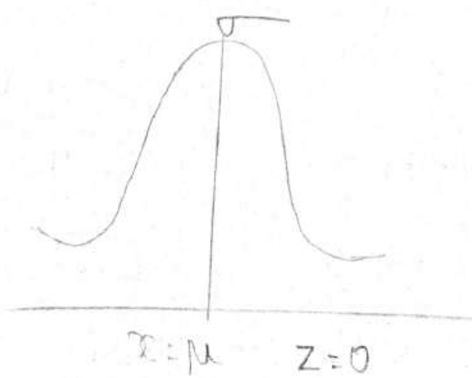






Standard Normal probability Distribution:

If X is a normally distributed random variable with μ (mean) and standard deviation σ then $Z = \frac{X - \mu}{\sigma}$ is called standard normal probability distribution.



1. Find the area under the standard normal curve which is lie in the

i) To right of $z = 2.70$

ii) To left of $z = 1.73$

iii) To right of $z = -0.66$

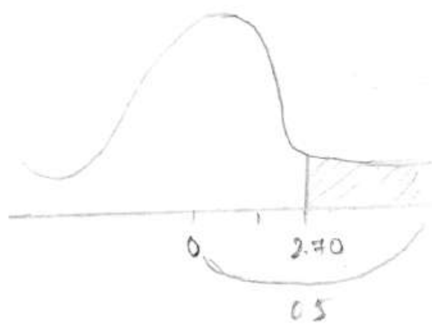
iv) To left of $z = -1.88$

v) Between $z = -0.90$ & $z = -1.85$

vi) Between $z = -1.45$ & $z = 1.45$

vii) Between $z = -0.90$ & $z = 1.58$

i) right $z = 2.70$

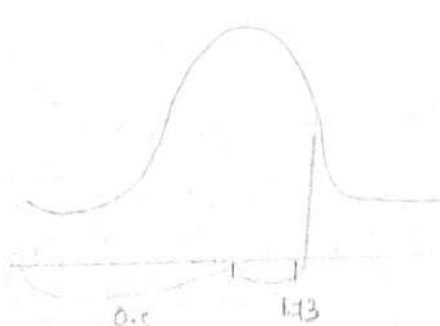


$= 0.5 - P(0 < z < 2.70)$

$= 0.5 - 0.4965$

$= 0.0035$

ii) left $z = 1.73$

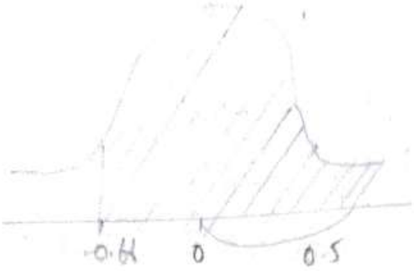


$= 0.5 + P(0 < z < 1.73)$

$= 0.5 + 0.4582$

$= 0.9582$

iii) Right $z = -0.66$



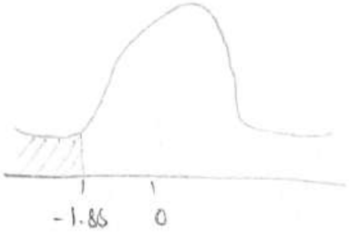
$= P(-0.66 < z < 0) + 0.5$

$= P(0 > z > 0.66) + 0.5$

$= 0.2454 + 0.5$

$= 0.7454$

iv) Left $z = -1.88$



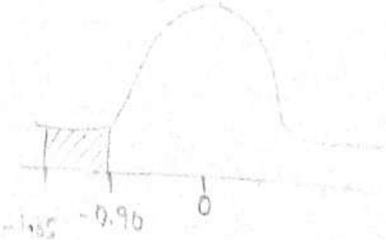
$= 0.5 - P(-1.88 < z < 0)$

$= 0.5 - P(0 > z > 1.88)$

$= 0.5 - 0.4699$

$= 0.0301$

v) Between $z = -0.90$ & $z = -1.85$



$P(-1.85 < z < 0) - P(-0.90 < z < 0)$

$= P(0 > z > 1.85) - P(0 > z > 0.90)$

$= 0.4678 - 0.3159 = 0.1519$

vi) Between $z = -1.45$ & $z = 1.45$

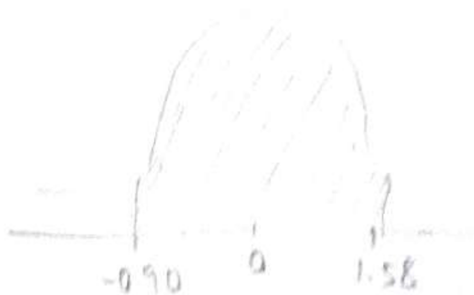
$= p(-1.45 < z < 0) + p(0 < z < 1.45)$

$= p(0 > z > 1.45) + p(0 < z < 1.45)$

$= 0.4265 + 0.4265$

$= 0.853$

vii) Between $z = -0.90$ & $z = 1.58$



$p(-0.90 < z < 0) + p(0 < z < 1.58)$

$= p(0 > z > 0.90) + p(0 < z < 1.58)$

$= 0.3159 + 0.4429$

$= 0.7588$

2. Assume that the mean of height of soldiers tobe
With Variance 27. How many Soldiers regiment
of 1000 Can be expressed expected to be over
182 cm

$\mu = 172$

Variance $= S.D = \sigma = 27$

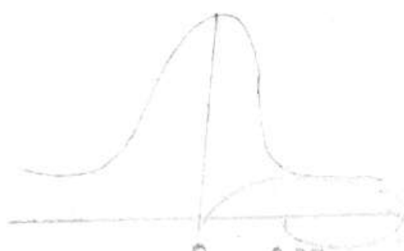
$x = 182$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{182 - 172}{27}$$

$$= \frac{10}{27}$$

$$= 0.37$$



$= 0.5 - p(0 < Z < 0.37)$

$= 0.5 - 0.1443$

$= 0.3557$

Number of Soliders expected to be over 182 cm in a regiment of 1000

$\rightarrow 1000 \times 0.3557$

$= 355.7$

3 Suppose the height of individual of a College are normally distribution with mean 160cm and S.D = 10cm we can determine the probability that a randomly selected person is

i) Above 180cm ii) Below 150cm.

Sol

$\mu = 160 \quad \sigma = 10$

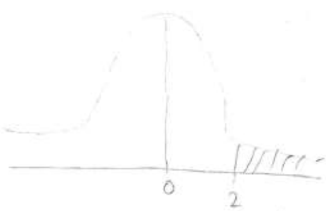
$$Z = \frac{X - \mu}{\sigma}$$

i) Above 180cm $(X = 180)$

$$Z = \frac{180 - 160}{10}$$

$$= \frac{20}{10}$$

$$= 2$$



$= 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772$

$= 0.0228$

ii) Below 150cm $(X = 150\text{cm})$

$$Z = \frac{150 - 160}{10}$$

$$= \frac{-10}{10}$$

$$= -1$$

$= 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413 \quad = 0.159$

4. The marks Obtained by a large group of Student in a final examination in a Statistics have a mean 58 and S.D 8.5. Assuming that this marks are approximately normally distributed. What percentage of the Student expected to have obtain marks from 60 to 69 both increased.

Sol

$\mu = 58 \quad \sigma = 8.5$

When, $x = 60$

The Standard normal variate is

$$Z = \frac{X-\mu}{\sigma}$$

$$= \frac{60-58}{8.5}$$

$$= \frac{2}{8.5}$$

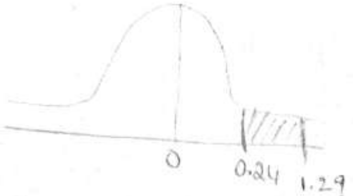
$$= 0.24$$

When $x = 69$

$$Z = \frac{69-58}{8.5}$$

$$= \frac{11}{8.5}$$

$$= 1.29$$



$P(60 < Z < 69)$

$= P(0.24 < Z < 1.29)$

$= P(0 < Z < 1.29) - P(0 < Z < 0.24)$

$= 0.4015 - 0.0948$

$= 0.3067.$ $\quad 100 \times 0.3067 = 30.67.$

5. The life of a certain kind of electronic device as a mean of 300 hours and a standard deviation of 25 hrs. Assuming that the distribution of life time which are measured to the nearest hrs can be approximated closely with a normal curve.

i) Find the probability that any of this devices will have a life time of more than 350 hours

ii) What percentage will have life time from 220 to 260 hrs.

Sol $\mu = 300$ hrs $\sigma = 25$ hrs $x = 350$

The standard normal variate is

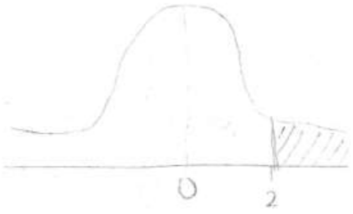
$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{350 - 300}{25}$$

$$= \frac{50}{25}$$

$$= 2$$

$p(x > 350) =$



i) Probability that life time is more than 350 hrs.

$$= p(X > 350)$$

$$= p(Z > 2)$$

$$= 0.5 - p(0 < Z < 2)$$

$$= 0.5 - 0.4772$$

$$= 0.0228$$

ii) When $x = 220$

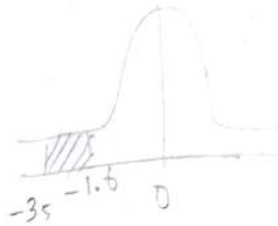
$$Z = \frac{220-300}{25}$$

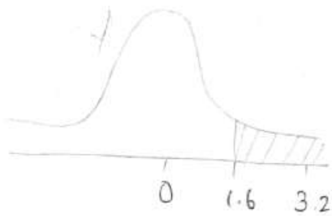
$$= -3.5$$

When $x = 260$

$$Z = \frac{260-300}{25}$$

$$= -1.6$$





$$P(220 < x < 260) = P(-3.2 < Z < -1.6)$$

$$= P(1.6 < Z < 3.2)$$

$$= P(0 < Z < 3.2) - P(0 < Z < 1.6)$$

$$= 0.4993 - 0.4452$$

$$= 0.054$$

$\therefore$ 5.41% electronic devices will have life time between 220 and 260 hrs.

6. Let X be normally distributed with mean $\mu = 8$ and standard deviation 4. Find i) $P(5 \le X \le 10)$ ii) $P(10 \le X \le 15)$ iii) $P(X \ge 15)$ iv) $P(X \le 5)$

Sol $\mu = 8$, $\sigma = 4$

The standard normal variate is

i) When $X = 5$

$$Z = \frac{X-\mu}{\sigma}$$

$$= \frac{5-8}{4}$$

$$= \frac{-3}{4}$$

$$= -0.75$$

When $x=10$

$$z=\frac{x-\mu}{\sigma}$$

$$=\frac{10-8}{4}$$

$$=\frac{2}{4}$$

$$=0.5$$

$$P(5\le x\le 10)=P(-0.75<z<0.5)$$

$$=P(-0.75<z<0)+P(0<z<0.5)$$

$$=P(0<z<0.75)+P(0<z<0.5)$$

$$=0.2734+0.1915$$

$$=0.4649.$$

ii) When $x=10$

$$z=\frac{x-\mu}{\sigma}$$

$$=\frac{10-8}{4}$$

$$=\frac{2}{4}$$

$$=0.5$$

When $x=15$

$$z=\frac{x-\mu}{\sigma}$$

$$=\frac{15-8}{4}=\frac{7}{4}\quad z=1.75$$

$$P(10\le x\le 15)=P(0.5<z<1.75)$$

$$=P(0<z<1.75)-P(0<z<0.5)$$

$$=0.4599-0.1915$$

$$=0.2684$$

iii) $P(x\ge 15)=P(z>1.75)$

$$=0.5-P(0<z<1.75)$$

$$=0.5-0.4599$$

$$=0.0401$$

iv) $p(x \leq 5) = p(z < -0.75)$

$= 0.5 - p(-0.75 < z < 0)$

$= 0.5 - p(0 < z < 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

7. In a distribution exactly normal 7% of the items are under 35 and 39% are under 63. What are the mean and S.D of the distribution

Sol The standard normal variate is

$$z = \frac{x-\mu}{\sigma}$$

Let $z = z_1$

when $x = 35$

$z = z_2$

when $x = 63$

$p(z_1 < z < 0) = 0.43$ from the table

$z_1 = -1.48$

$$z_1 = \frac{x-\mu}{\sigma}$$

$$-1.48 = \frac{35-\mu}{\sigma}$$

$-1.48\sigma = 35-\mu \rightarrow (1)$

$p(0 < z < z_2) = 0.39$ from the table

$z_2 = 1.23$

$$z_2 = \frac{x-\mu}{\sigma}$$

$$1.23 = \frac{63-\mu}{\sigma}$$

$$1.23\sigma = 63-\mu \rightarrow (2)$$

Eq (1) - (2)

$$-1.48\sigma = 35-\mu$$

$$1.23\sigma = 63-\mu$$

$$-2.71\sigma = -28$$

$$\sigma = \frac{28}{2.71}$$

$$\sigma = 10.33$$

Sub in (1)

$$-1.48\sigma = 35-\mu$$

$$-1.48(10.33) = 35-\mu$$

$$-15.28 = 35-\mu$$

$$\mu = 35+15.28$$

$$\mu = 50.28$$

8 Students of a class were given an aptitude test. There marks were found to be normally distribution with mean 60 and S.D = 5. What % Student scored i) More than 60 marks
ii) Less than 56 marks
iii) Between 45 and 65 marks in percentage.

Sol

$\mu = 60 \quad \sigma = 5$

The Standard normal Variate is

$$Z = \frac{X-\mu}{\sigma}$$

ii) When $X = 56$

$$Z = \frac{56-60}{5}$$

$$= -4/5 = -0.8$$

$$P(X < 56) = P(Z < -0.8)$$

$$= 0.5 - P(0 < Z < 0.8)$$

$$= 0.5 - 0.2881 = 0.2119$$

When $x = 45$

$$Z = \frac{45-60}{5} = -15/5 = -3$$

When $x = 65$

$$Z = \frac{65-60}{5} = 5/5 = 1$$

$$P(45 < X < 65) = P(-3 < Z < 1)$$
$$= P(-3 < Z < 0) + P(0 < Z < 1)$$
$$= P(0 < Z < 3) + P(0 < Z < 1)$$
$$= 0.49865 + 0.3413$$
$$= 0.84$$

ii) When $x = 60$

$$Z = \frac{X-60}{5} = \frac{60-60}{5}$$

$$Z = 0$$

$$P(X > 60) = P(Z > 0)$$
$$= 0.5 - P(0 < Z < 0)$$
$$= 0.5 - 0.0000$$
$$= 0.5$$

9. The customer accounts of a certain departmental Store have an average balance of rupees 120 and SD 40. Assuming that the account balance are normally distributed.

1) What population of Accounts over ₹150
2) What population of account is between ₹100 & ₹150
3) What population of account is between ₹60 & ₹90.

Given $\mu = 120$ $\sigma = 40$

$$Z = \frac{X - \mu}{\sigma}$$

$X = 150$

$$Z = \frac{150-120}{40} = \frac{30}{40}$$

$Z = 0.75$

$P(X > 150) = P(Z > 0.75)$

$= 0.5 - P(0 < Z < 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

When $X = 60$

$Z = \frac{X - \mu}{\sigma} = \frac{60 - 120}{40} = \frac{-60}{40} \quad Z = -1.5$

When $X = 90$

$Z = \frac{X - \mu}{\sigma} = \frac{90 - 120}{40} = \frac{-30}{40} \quad Z = -0.75$

$P(60 < X < 90) = P(-1.5 < Z < -0.75)$

$= P(-1.5 < Z < 0) - P(0 < Z < -0.7)$

$= P(0 < Z < 1.5) - P(0.75 > Z > 0)$

$= 0.4332 - 0.2734$

$= 0.1598$

When $X = 100$

$Z = \frac{100 - 120}{40}$

$= \frac{-20}{40}$

$Z = -0.5$

When $X = 150$

$Z = \frac{150 - 120}{40}$

$= 30/40$

$= 0.75$

$P(100 < X < 150) = P(-0.5 < Z < 0.75)$

$= P(-0.5 < Z < 0) + P(0 < Z < 0.75)$

$= P(0 < Z < 0.5) + P(0 < Z < 0.75)$

$= 0.1915 + 0.0.2734$

$= 0.4649.$

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Additional Resources:

https://www.youtube.com/watch?v=a7FjKBYBc3o .

Practice Questions:

Section – A

1. Define Independent Event?
2. Explain the properties of Binomial Distribution?
3. Define Poisson frequency distribution?
4. Explain about Random experiment?
5. Given a normal curve with $\mu = 25.3$ and $\sigma = 8.1$.Find the area under the curve between 20.6 and 29.1.
6. Write the formulas for Binomial distribution.
7. Write the formulas for Normal distribution.
8. Write the formulas for Poisson distribution.
9. Define Bernoulli trial.
10. Two coins are tossed simultaneously what is the probability of getting a head and a tail.
11. A perfect die is tossed twice. Find the probability of getting a total of 9.
12. For a binomial distribution with a parameters x=5,P=0.03, Find the probability of getting exactly 3 failure.
13. A random variable X follows a poisson distribution such that P(X=2)=P(X=1). Find P(X=0).
14. If a pair of dice is throwned. Find the probability that the sum is neither 7 or 11.
15. Define Poisson distribution.
16. Define Normal distribution.
17. Define Binomial distribution.
18. Define rules of addition.

Section – B

1. Explain about Mutually exclusive event and exhaustive event?
2. Describe the Fundamental rules of Probability?
3. An integer is choosen as random out of the integer from 1 to 100.What is the Probability that is (i) Multiple of 5 (ii) Divisible by 7 (iii) Greater than 70.
4. If 10% of the screws produced by an automatic machine is defective. Find the probability that of 20 screws selected at random. There are
   (i) Exactly 2 defective (ii)Atmost 3 defective (iii) Atleast 2 defective. Find also the mean, variance and skewness of the number of defective screws.
5. Students of a class were given an aptitude test. These marks were found to be normally distribution with mean 60 and standard deviation 5 what percentage student scored (i) More than 60 marks
   (ii) Less than 56 marks (iii) Between 45 and 65 marks in percentage.

6. A bag contains 4 white and 6 black balls. Two balls are drawn at random. What is the probability that (i) both are white (ii) both are black (iii) one white and one black.
7. 10 coins are tossed simultaneously, Find the probability of getting (i) atleast 7 head (ii) atmost 7 head (iii) Exactly 7 head.
8. A mean of binomial distribution is 5 and standard deviation is 2 Determine the probability distribution.
9. The mean and variance of binomial variant are 8 and 6.Find $P(X \geq 2)$.
10. If X is a poisson random variable such that P(X=1)=0.3 and P(X=2)=0.2. Find P(X) = 0.

Section – C

1. State and Prove Baye's Theorem.
2. Explain briefly on Normal distribution.
3. Explain briefly on Binomial distribution.
4. Explain briefly on Poisson distribution.
5. Explain about the probability distribution function.
6. Let X be a normally distributed with mean $\mu = 8$ and standard deviation is 4. Find $(i)\ P(5 \leq X \leq 10)(ii)P(10 \leq X \leq 15)\ (iii)P(X \geq 15)(iv)(X \geq 5)$
7. A husband and wife appear in a interview for two vacancies in the same post the probability of husband selection is $\frac{1}{7}$ and that of wife selection is $\frac{1}{5}$. What is the probability that
   (i) both of them will be select (ii) only one of them will be select and (iii) none of them will be select.
8. The lifetimes of certain kinds of electronic devices have a mean of 300 hours and standard deviation of 25hours. Assuming that the distribution of these lifetimes which are measured to the nearest hour, can be approximated closely with a normal curve
   (a) Find the probability that any one of these electronic devices will have a lifetime of
   (b) more than 350 hours. (b) What percentage will have
   lifetimes of 300hours or less? (c) What percentage will have lifetimes from 220 or 260 hours?

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