MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

E-NOTES

SUBJECT: CONDENSED MATTER PHYSICS

SUBJECT CODE: DPH31

UNIT : 3

SYLLABUS:

Monoatomic lattices - Lattice with two atoms per primitive cell - First Brillouin zone - Group and phase velocities - Quantization of lattice vibrations - Phonon momentum - Inelastic scattering by phonons - Einstein‘s model and Debye‘s model of specific heat.

UNIT - III.

WIEDMANN FRANZ LAW

Statement

The ratio between the thermal conductivity and electrical conductivity of a metal is directly proportional to the absolute temperature of the metal.

$$\frac{K}{\sigma} \propto T.$$

$$\frac{K}{\sigma} \propto T \quad \therefore \frac{K}{\sigma} = LT$$

Where, L is constant it is called a Lorentz's Number whose value is $L = 2.44 \times 10^{-8} \, W\Omega K^{-2}$.
(or)
$W\Omega/K^2$

Proof

i) By classical theory.

We know electrical conductivity from classical theory.

$$\sigma = \frac{ne^2\tau}{m}$$

n = free e^- density.
e = charge of e^-
τ = torque
m = mass of e^-.

$\sigma = \frac{ne^2\tau}{m}$

$K = \frac{nv^2\tau k_B}{2}$

Thermal conductivity. (from C.T)

$$k = \frac{nv^2 K_B \tau}{2}$$

n : free e^- density

v = Velocity.

K_B = Boltzmann Const

$$\frac{K}{\sigma} = \frac{\frac{nv^2 K_B \tau}{2}}{\frac{ne^2 \tau}{m}}$$

$$= \frac{nv^2 K_B \tau}{2} \times \frac{m}{ne^2 \tau}$$

$$\boxed{\frac{K}{\sigma} = \frac{v^2 K_B m}{2e^2}} \quad \text{——— ①.}$$

$$= \frac{1}{2} mv^2 . \frac{K_B}{e^2}.$$

We. know.

kinetic energy of an e^- = $\frac{1}{2} mv^2$

$= \frac{3}{2} K_B T.$

Sub this eqn in ①

$$\frac{K}{\sigma} = \frac{3}{2} \frac{K_B^2 T}{e^2}.$$

$$\boxed{\frac{K}{\sigma} = L T}$$

$$\therefore L = \frac{3}{2} \frac{K_B^2}{e^2}.$$

$K_B = 1.38 \times 10^{-23}$ J/K.

$e = 1.60 \times 10^{-19}$ J.

$$L = \frac{3}{2}\left(\frac{1.38 \times 10^{-23}}{1.602 \times 10^{-19}}\right)^2$$

$$L = 1.12 \times 10^{-8}\ W\Omega K^{-2}$$

It is found that the classical value of Lorentz's Number, is only half of the experimental value i.e, $2.44 \times 10^{-8}\ W\Omega K^{-2}$.

The discrepancy in the experimental and the theoretical value of "L" is the failure of classical theory. This discrepancy can be rectified by quantum theory.

By quantum theory.

In quantum theory, the mass of the e (m), is replaced by the effective mass (m^*).

The electrical conductivity by Quantum theory

$$\sigma = \frac{ne^2\tau}{m^*}$$

In Thermal conductivity, rearranging the expression, and sub the electronic specific heat.

$$K = \frac{\pi^2}{3}\frac{nK_B^2\tau}{m^*} \cdot T. \qquad \frac{n\pi^2 K_B^2 \tau}{3\, m^*}\ T$$

$$\frac{k}{\sigma} = \frac{\frac{\pi^2}{3}\left(\frac{n k_B^2 \tau}{m^*}\right) T}{\frac{n e^2 \tau}{m^*}}$$

$$= \frac{\pi^2}{3}\frac{n k_B^2 \tau}{m^*} . T \Big/ \frac{n e^2 \tau}{m^*}$$

$$= \frac{\pi^2 n k_B^2 \tau . T}{3 m^*} \times \frac{m^*}{n e^2 \tau}$$

$$= \frac{\pi^2 k_B^2 T}{3 e^2}$$

$$= \frac{\pi^2}{3}\left[\frac{k_B}{e}\right]^2 . T$$

$$\boxed{\therefore L = \frac{\pi^2}{3}\left[\frac{k_B}{e}\right]^2 .}$$

$k_B = 1.38 \times 10^{-23}$ J/k

$e = 1.602 \times 10^{-19}$ J

$\pi = 3.14$

$$= \frac{(3.14)^2}{3} \frac{(1.38 \times 10^{-23})^2}{(1.602 \times 10^{-19})^2} .$$

$$\boxed{L = 2.44 \times 10^{-8} \; W \Omega K^{-2}}$$

This Quantum theory verifies Wiedmann franz law and has good agreement with the experimental value of Lorentz's Number.

Hall effect

Measurement of conductivity will not determine whether the condition due to electron or holes and therefore will not distinguish between p type and n-type semi conductors.

Therefore hall effect is used to distinguish b/w the two types of carriers and their carrier densities and is used to determine the mobility of charge carriers.

Hall effect :

(A) When a conductor (metal or semi conductor) carrying current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and the magnetic field.

This phenomenon is known as hall effect and the generated voltage is called as hall voltage.

Hall effect in n-type semiconductor.

Let us consider an n-type material to which the current is allowed to pass along x-direction from left to right and the magnetic field is applied in z-direction. As a result hall voltage is produced in y-direction. as shown in figure A.

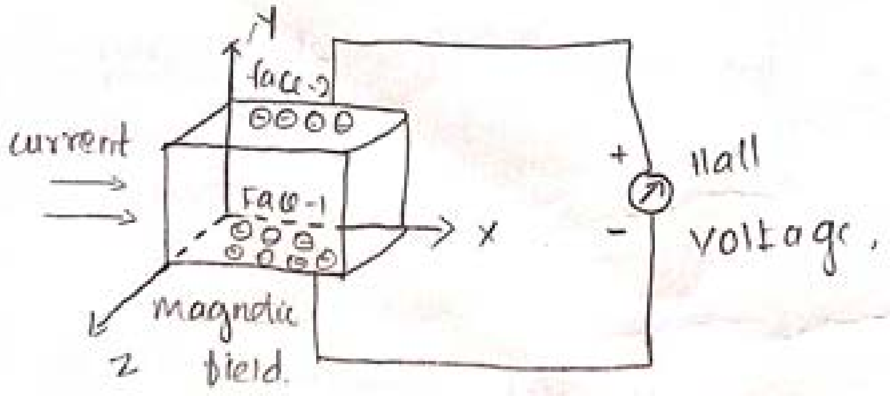


fig ⓐ

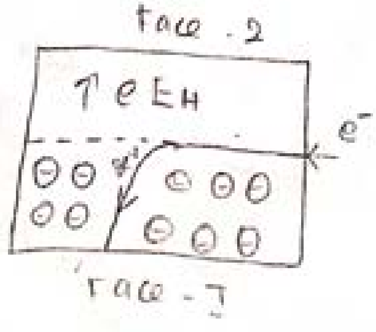


fig (b)

Since the direction of current is from left to right the electron moves from right to left in x-direction, as shown in figure b.

Now due to the magnetic field applied the electrons move towards downward direction with the Velocity V and causes the negative charge to accumulate at face I of the material as shown in figure a & b.

Therefore a potential difference is established b/w face-I and face-II of the specimen which gives rise to field E_H in the -ve Y-direction Here the force due to potential difference is equal to $-E_H$

The force due to potential difference

$= -eE_H$ ——— ①

The force due to magnetic field

$= -BeV \longrightarrow$ ②

At equilibrium eqn ① = eqn ②

$$-eE_H = -BeV$$

$$E_H = +BV \longrightarrow ③$$

We know the current density $J(x)$ in the x - direction is

$$\boxed{J_x = -n_e eV}$$

$$V = -\frac{J_x}{n_e e} \longrightarrow ④$$

sub ④ in ③ $n_e \rightarrow$ electron density

$$E_H = +B\left(-\frac{J_x}{n_e e}\right)$$

$$= -\frac{BJ_x}{en_e} \longrightarrow ⑤$$

$$E_H = R_H \cdot J_x \cdot B \longrightarrow ⑥$$

where $\underline{R_H}$ is known as Hall co-efficient given by $R_H = -\left(\frac{1}{n_e e}\right)$

The -ve sign indicates that the field is developed in the -ve y direction.

Hall effect in P-type semiconductors.

Let us consider a P-type material for which the current is passed along x-direction from Left to right and magnetic field is applied along z-direction as shown in figure A.

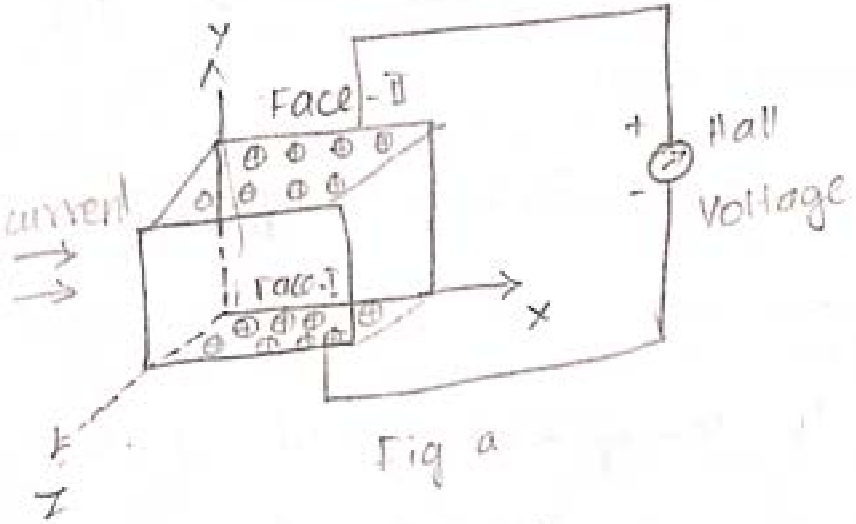


Fig a

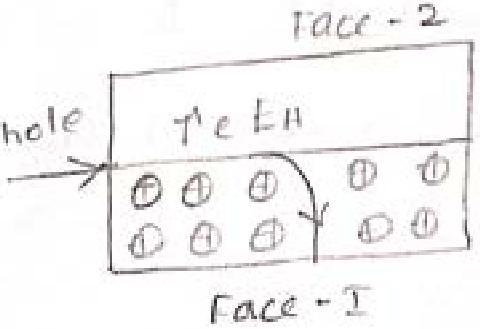


fig (b).

Since the delection of current is from left to right the holes will also move in the same direction as shown in fig (B).

Now due to magnetic field applied. The holes, move towards the downward direction with velocity 'v' and accumulate at the farce (1) as shown in fig (A) and fig (b).

A potential difference is established between face (1) and face (2) in the +ve of y- direction

force due to potential difference

$= eE_H$ —— ⑦.

Force due to magnetic field $= BeV$ —— ⑧

Since hole is considered to be an e^- with same mass and +ve charge -ve sign is not in included

At equilibrium

eqn ⑦ = eqn ⑧

$$eE_H = BeV$$

$$E_H = BV \longrightarrow ⑨$$

We know

$V = \frac{J_x}{n_h e}$ ——— ⑩

Where n_h => hole density substituting eqn ⑩ in eqn ⑨ we get.

$$E_H = \frac{BJ_x}{n_h e}$$

$$E_H = R_H J_x B$$

Where

$$R_H = \frac{1}{n_h e} \longrightarrow ⑪$$

eqn ⑪ represents the hall co-efficient and the +ve sign indicates that the hall field is developed in the +ve y-directio

R'' Hall-co-efficient interms of hall Voltage :-

In the thickness of the sample is t, and the voltage developed in V_H, Then

Hall voltage $V_H = E_H . t \longrightarrow ⑫$

Sub the eqn ⑥ in eqn ⑫ we have

$$V_H = R_H J_x B t \longrightarrow ⑬$$

If b is the width of the sample than

Area of the sample $= b.t$

$\therefore$ current density $J_x = \frac{I_x}{bt} \longrightarrow ⑭$

Substituting eq ⑭ in eq ⑬ we get

$$V_H = \frac{R_H I_x B \cancel{t}}{b\cancel{t}}$$

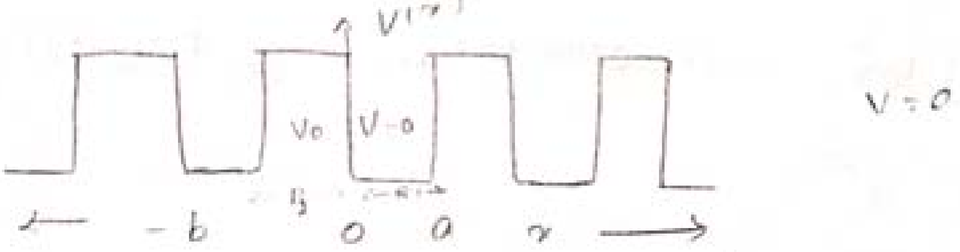
$$V_H = \frac{R_H I_x B}{b} \qquad R_H = \frac{V_H b}{I_x B}$$

Hall co-efficient, $R_H = \frac{V_H b}{I_x B}$.

Behaviour of an electron in a periodic potential (kronig - penny model)

kronig and penny heated a simple set example for one dimensional periodic potential.

In this model it is assumed that the potential energy of an electron has the form of a periodic away of square well as shown in figure.



Here we have two regions viz.

Region I

In this region b/w the limits $0 < x < a$ the potential energy is zero and have hence the e- is assumed to be a free particle.

The 1- dimensional schrodinger wave eqn of a free particle is.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-0)\psi = 0$$

$$\boxed{\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0} \longrightarrow ①$$

Where

$$\alpha^2 = \frac{2m}{\hbar^2} E$$

Region (II)

In this eqn b/w the limit $-b < x < 0$. The potential energy of the e^- is V_0

The one dimensional schrodinger wave eqn is,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi = 0$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \longrightarrow ②$$

Where

$$\beta^2 = \frac{2m}{\hbar^2}(V_0 - E) \quad [\text{since } E < V_0).$$

For both, the region the appropriate soln suggested by block is g of the form,

$\psi(x) = e^{ikx} u_k((x))$ ——③

Differentiating eqn ③ and sub it in eqn ① and eqn ② & Then further solving it under the boundary condition we get

$$P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos k a \longrightarrow ④$$

Where

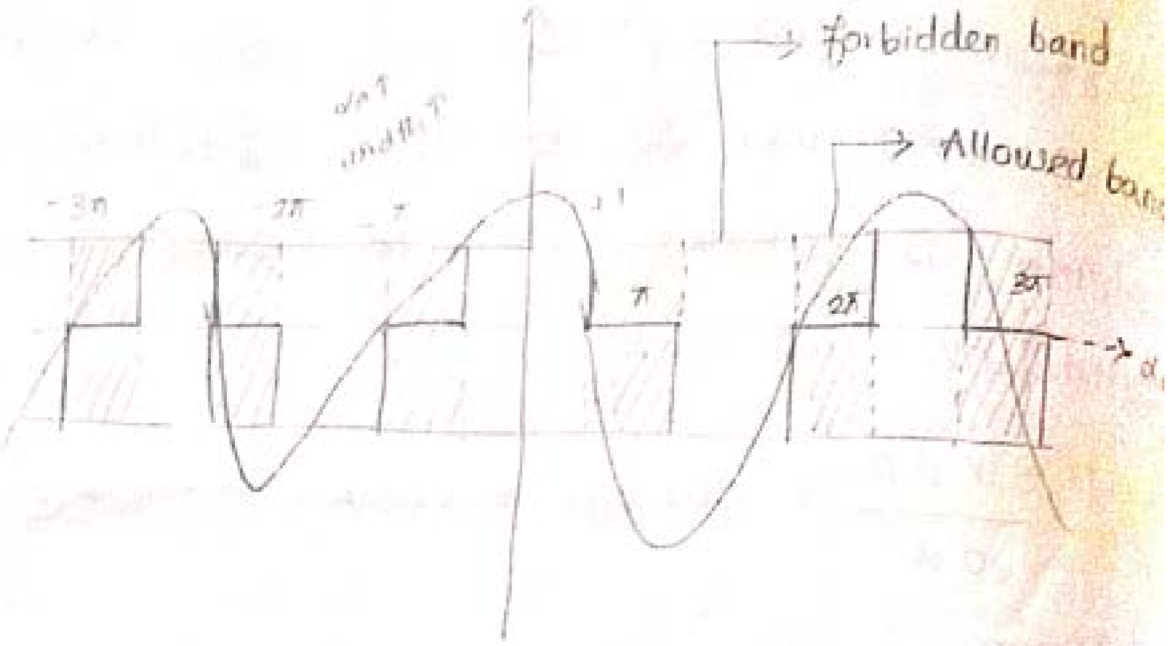
$P = \frac{m V_0 b a}{\hbar^2}$ is called a scattering power of the potential barrier which is the measure of the strength with which is the e^- are attracted by the positive ions

In eqn ④ there are only 2 variable i.e α and k . We known cosk a can take values only from -1 to +1

∴ L.H.S of eqn ④ must be also fall in this range

A plot is made b/w the L.H.S of eqn ④ and α a for a value of $P = \frac{3\pi}{2}$

(arbitrary) as shown in figure.



$$P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

conclusions:

from the figure.

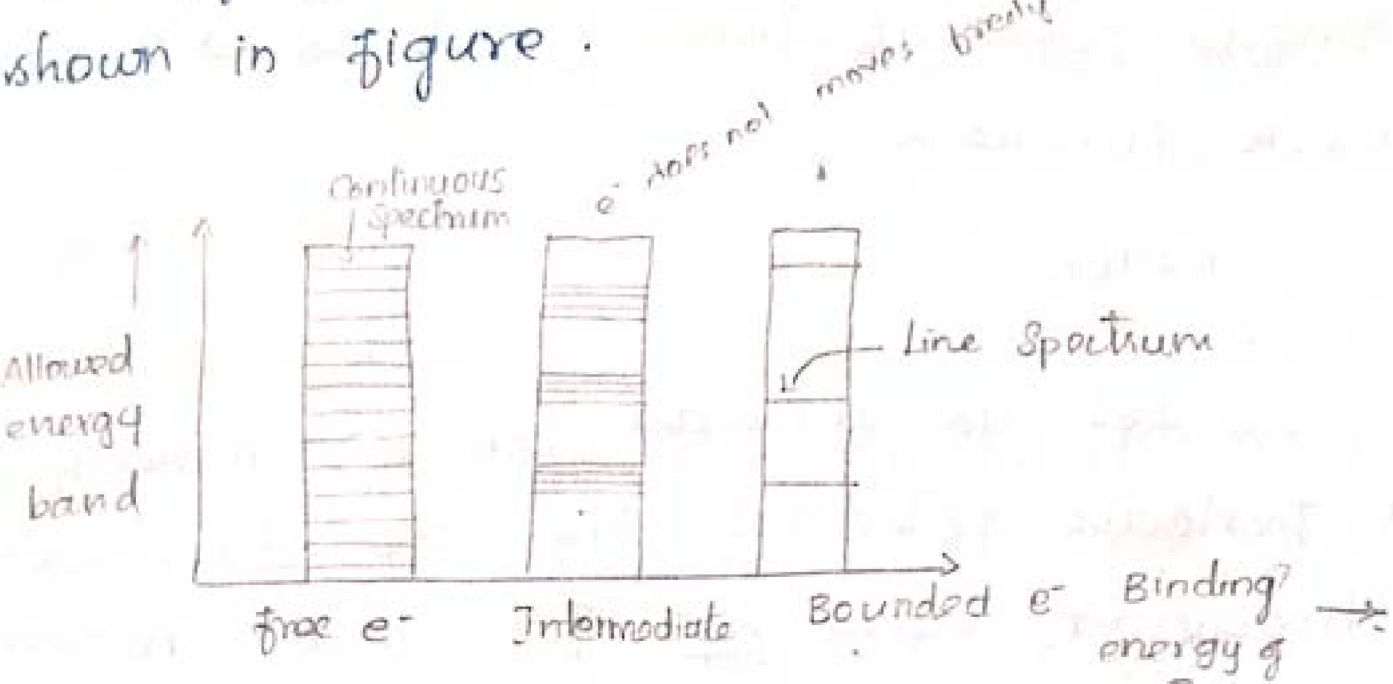
i) The energy spectrum has a no. of allowed energy bands denoted by solid horizontal line separated by forbidden band gaps denoted by dotted lines

ii) The width of allowed energy band (shaded portion) increases with the increase in αa

iii) When P is increased, the binding energy of the e^- with the lattice points is also increased   P↑ BE of e ↑ P↑

∴ The e^- will not be able to move freely and hence the width of the allowed energy band is decreased.

Especially for $p \to \infty$, then allowed energy band becomes infinitely narrow and the energy spectrum becomes a line spectrum as shown in figure.



iv) When p is decreased, the binding energy of the e- decreases and Thus it moves freely over the lattice points and hence we get a wide range of allowed energy levels as shown in figure

v) Thus by varying p from zero to infinity we get the energy spectra of all ranges.

⑧ 5M BLOCH THEOREM

It is a mathematical statement of an e^- wave function moving in a perfe Periodic potential. These functions are called block functions.

EXPLANATION

Let us consider an e^- moving in a periodic potential. The one dimensional schrodinger wave eqn for an e^- moving in a periodic potential shall be written as

$$\frac{d^2\psi}{dx^2}(x) + \frac{2m}{\hbar^2}[E-V(x)]\,\psi(x) = 0 \longrightarrow ①$$

Suppose the e^- moves along x-direction in a one dimensional crystal, then the potential energy of the e^- should satisfy the condition

$$V(x) = V(x+a) \longrightarrow ②$$

Where a is the periodicity of the Potential. The soln of eqn 1 is

$$\psi(x) = e^{ikx}\,u_k(x) \longrightarrow ③$$

Where $u_k(x) = u_k(x+a) \longrightarrow$ ④.

Here, e^{ikx} represents the plane wave and $u_k(x)$ represents the periodic functions and eqn 3 is called bloch theorem. and eqn 4 is called bloch function

PROOF:

If eqn ① has the soln with the property of eq ②,

We can write the property of the bloch functions. i.e, eqn ③ has

$$\psi(x+a) = e^{ik(x+a)} u_k(x+a).$$

(or)

$$\psi(x+a) = e^{ikx} \cdot e^{ika} \cdot u_k(x+a).$$

Since,

$$u_k(x+a) = u_k(x)$$

We can write the above eqn as

$$\psi(x+a) = e^{ikx} \cdot e^{ika} u_k(x).$$

from eqn ③.

$$\psi(x+a) = \psi(x)\, e^{ika} \longrightarrow ⑤.$$

(or).

$$\psi(x+a) = \Phi\,\psi(x) \longrightarrow ⑥$$

where $\boxed{\Phi = e^{iKa}}$

If $\psi(x)$ is a single valued then if $\psi(x)$ is a single valued function.

Then it can be expressed as

$$\psi(x+a) = \psi(x).$$

Thus bloch theorem is proved.

This eqn is similar to that of eqn ② and ④ . i.e, if the potential is the function of 'x' and 'a'. then the wave function is also a function of x and a.

FREE ELECTRON GAS IN 3D

(x) 10M

Let us consider, free that the electron confined to remain inside a cubical box of edge L. The potential energy of the e^- inside the box every where is taken as zero. The wave function $\psi_R(r)$ is described by the schrodinger eqn of the form.

$$\nabla^2 \psi_R(r) + \frac{2m}{\hbar^2} E_R\,\psi_R(r) = 0 \longrightarrow ⑦$$

Where

E_R is the Total energy of the e^- Here it is kinetic energy of the e^- in the state R. It can be shown that the Normalised wave function satisfying the eqn ① is given by.

$$\psi_R(r) = \left(\frac{8}{L^3}\right)^{1/3} \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right)$$

$$\cdot \sin\left(\frac{n_z \pi z}{L}\right) \longrightarrow ②.$$

Where

n_x, n_y, n_z are +ve integers. This is a standing wave solution.

However, it is more convenient to work with a plane travelling wave soln of the form.

$$\psi_R(r) = A e^{i\vec{R}\cdot\vec{r}}$$

$$= A e^{i(R_x x + R_y y + R_z z)}$$

Where.

$$R^2 = \left(\frac{2m}{\hbar^2}\right) E_R$$ and

A is arbitrary constant;

such wave soln must satisfy the following periodic boundary condition

$$\left.\begin{aligned} \psi(x+L, y, z) &= \psi(x, y, z) \\ \psi(x, y+L, z) &= \psi(x, y, z) \\ \psi(x, y, z+L) &= \psi(x, y, z) \end{aligned}\right\} \longrightarrow ③$$

These conditions obviously make the wave function to be periodic in x, y, z with period 'L'.

Applying the 1st Boundary Condition We have,

$e^{ik_x(x+L)+k_y y}$

$$\exp i\,[k_x(x+L)+k_y y+k_z z] =$$

$$= \exp i\,[k_x x+k_y y+k_z z]$$

(or)

$$K_x = 0, \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L} \cdots + \frac{2\pi n_x}{L} \rightarrow ④$$

$$k_y = \pm\frac{2\pi n_y}{L}$$

$$k_z = \pm\frac{2\pi n_z}{L}$$

where n_x, n_y, n_z are integers including zero.

$$K^2 = K_x^2 + K_y^2 + K_z^2 = \left(\frac{2\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

$$= \left(\frac{2\pi}{L}\right)^2 n^2$$

$$\boxed{\therefore \; n_x^2 + n_y^2 + n_z^2 = n^2}$$

Thus, the energy eigen values are given by

$$E_K = \frac{\hbar^2}{2m} K^2$$

$$= \frac{\hbar^2}{2m} (K_x^2 + K_y^2 + K_z^2) \longrightarrow ⑤$$

Where K_x, K_y, K_z are obtained according to conditions ④

Evidently, the energy spectrum consist of discrete energy levels which usually be very close to gether $(\sim 10^{-15}$ eVapar

Such energy levels are said to be quasi-continuous. These energy separation

equal No. of atoms
in a given

depends upon the size of the box.

To obtain the expression for A
we have.

$$\int_0^V \psi^*(r)\,\psi(r)\,dv = 1$$

Hence we get.

$$A = \left(\frac{1}{V}\right)^{1/2}.$$

Thus the Normalised wave function is

$$\psi_k(r) = \left(\frac{1}{V}\right)^{1/2} e^{i\vec{k}\cdot\vec{r}}.$$

Obviously $\vec{k}$ is a wavevector and k_x, k_y, k_z are the Components of $\vec{k}$.

The set of values of k_x, k_y, k_z defines a state of energy.

If m_s be the spin quantum Number of the e; then k_x, k_y, k_z, m_s define the energy state of the e^-.

The state of the system is defined by the quantum Number k_x, k_y, k_z, m_s. The e^- can have only two Spins $\pm 1/2$.

The e corresponding to Two possible orientation.

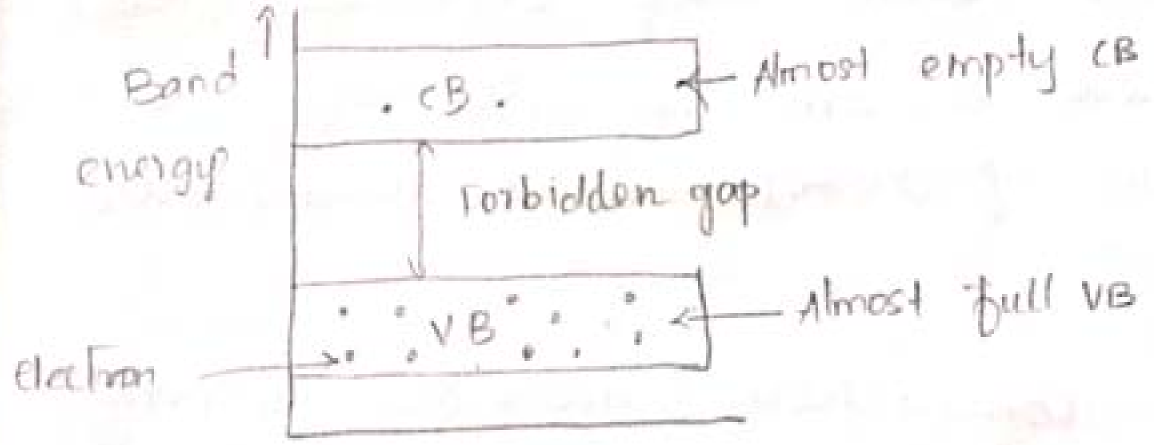
directions

Semi conductor

The Material which has electrical conductivity btween that of a conductor and that of an insulator is called as Semiconductor

Eg: silicon, germanium, graphite are some eg of semi conductors.

Semi conductors are the foundation of modern electronics including transistors, LEDS and solar cells etc..



In Semi Conductors, the forbidden gap between valence band and conduction band is very small. It has a forbidden gap of about 1 eV.

At low temperature. the Valence band is completely Occupied with e^- and conduction band is empty

Because the e^-s. in the valence ban does not have enough energy to move into conduction band.

∴ Semiconductor behaves as an insulat at low temperature.

However, at room temperature some of the e^-s in valence band gains enough energy in the form of heat and moves into a conduction band.

When the valence e^- moves into a conduction band. They becomes free e^-s.

These e^-s are not attached to the nucleus of a atom, so they moves freely.

The conduction band e^- are responsible for electrical conductivity. The measure of ability to conduct electric current is called as electrical conductivity.

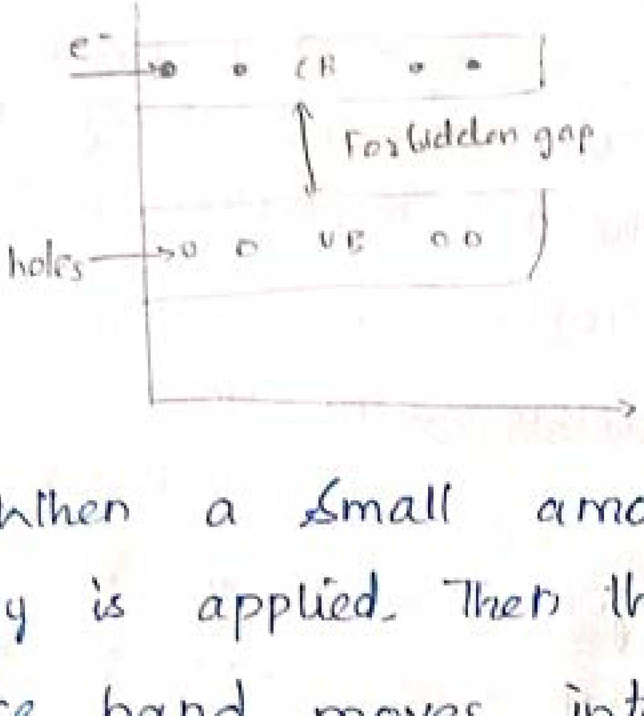
When the temp is goes on increasing, the No. of valence band e^- moving into conduction band is also increases.

This shows the electrical conductivity of the semi conductor increases with increase in temperature

HOLE

The absence of e⁻ in a particular place in an atom is called as hole.

Hole is the electric charge carrier which has positive charge. The electric charge of hole is equal to electric charge of electron, But have opposite polarity.



When a small amount of external energy is applied. Then the e⁻s in the valence band moves into conduction band and leaves a vacancy in valence band. This vacancy is called as hole.

Intrinsic Semi conductor

Intrinsic carrier concentration

In intrinsic semi conductor, when the valence e's broke the covalent bond and jumps into the conduction band, two types of charge carriers get generated. They are free e's and holes.

The No. of e's per unit volume in the conduction band or the Number of holes per unit volume in the valence band is called Intrinsic carrier concentration. The No. of e⁻ per unit volume in the CB is e⁻ carrier. The No. of hole per unit volume in the VB is called as hole-carrier concentration.

In an intrinsic semi conductor, the no. of e- generated in the C.B is equal to the No. of holes generated in the VB. Hence the e⁻-carrier is equal to hole-carrier

It can be written as.

$$n_i = n = p$$

where.

n = e⁻ carrier concentration

P = hole carrier concentration.

ni = intrinsic carrier con. The hole concentration in the valence band is given as

$$P = N_v e - \frac{(E_F - E_V)}{K_B T}$$

The e^- con. in the conduction Band is given as

$$n = N_c e - \frac{(E_C - E_F)}{K_B T}$$

Where KB is the Boltzmann constant

The T is the absolute Temperature of intrinsic semiconductor.

Nc is the effective density of states in conduction band.

Nv is the effective density of states in valence band.

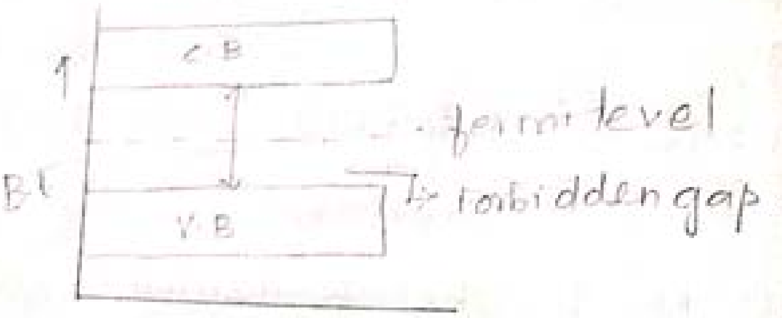
EF is the fermi energy.

Ec is the conduction energy.

FERMI - LEVEL IN INTRINSIC SEMICONDUCTOR

The probability of occupation of energy levels in valence band & CB is called fermi level. At absolute zero temperature intrinsic semiconductor acts as perfect insulator. However as the temperature increases free e- and holes get generated

In intrinsic or pure semiconductor the No. of holes in valence band is equal to the no. of e^- in the [illegible] the Probability of occupation of energy Levels in CB and VB are equal. ∴ The fermi-Level for the intrinsic semi conductor Lies in The middle of forbidden band



Fermi-level in the middle of forbidden band indicates equal concentration of free electrons and holes

The hole con. in the Valence bond is given as $P = N_V e^{-\frac{(E_F - E_V)}{k_B T}}$.

The electron concentration in the CB is

$$n = N_c e^{-\frac{(E_C - E_F)}{k_B T}}.$$

Where k_B is The Boltzmann constant

T is the absolute temp. of the intrinsic Semi conductor

N_c is the effective density of state in CB, N_v is the effective density of VB.

The No. of e^- in the CB is depends on effective density of states in the CB and ne distance of fermi level from the CB

The No. of holes in the Valence band is depends on effective density of states in The valence band and the distance of fermi level from the VB

For an intrinsic semiconductor, the e^-. carrier concentration is equal to the hole-carrier concentration.

It can be written as.

$$p = n = n_i$$

Where

p = hole-carrier con.

n = e^-- carrier con.

n_i = intrinsic carrier con.

The fermi level for intrinsic semiconductor is given as.

$$E_F = \frac{E_c + E_v}{2}.$$

Where E_F is the fermi level

E_c is the conduction band

E_v is the valence band

∴ The fermi level in an intrinsic semiconductor lies in the middle of the forbidden gap.

DE HASS-VAN ALPHEN EFFECT.

In case of free electron gas. We have calculated the paramagnetic Suceptibility due to the intrinsic magnetic moment associated wil

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MOBILITY, DRIFT VELOCITY AND CONDUCTIVITY OF INTRINSIC SEMI CONDUCTOR

Due to thermal agitation and lattice vibrations, some of the covalent bond in a semi conductor are broken and some e̅s. become free.

When an electric field is applied, the electrons drift towards the positive electrode and hence, constitute the current.

slide

Produce.

Sillry Absence of an e̅s from a bond, known as hole. Constitutes the net positive charge. This hole Move towards the negative electrode. hence constitute the current.

Thus, Due to Application of potential difference btw both ends of the crystal. charge carriers attain a drift Speed = V_d which is proportional to the electric field Strength

F

and is difference for different semi conducting materials and for different type of charge carriers

$$V_d \propto F$$

$$\boxed{V_d = \mu F}$$ $J_n \,.\, n_e V.$

Where μ, is the mobility of charge carrier define as the velocity of charge carrier produced due to unit electric field strength.

It should be noted here that in a pure semiconductor under the influence of an electric field, electrons move in a nearly empty conduction band. Where as, holes move in a nearly filled valence band. $S.C \xrightarrow{EF} e^{-}s \xrightarrow{empty} CB \;:\; S.C \xrightarrow{EF} '' \xrightarrow{filled} VB.$

I.e, An electron in conduction band is subjected to different conditions compared to a holes in valence band.

Thus, properties assigned to a hole. eg. mass, mobility etc. will not be the same as those of an e^- in conduction band.

∴ The type of drift of e^- and of holes under the action of an applied force are different.

μ_n the e^- mobility at any tempera
electron
will be different. (greater > from the Mobility, μ_p of holes .

If n is the density of free e^-s in CB. of Semi conductor. Then net charge per unit volume available (Total) for the conduction of e^- current will be "ne". ne.

= ne.

The drift current density J_n due to e^- is defined as the charge flowing across unit area of cross section per unit time due to the drift under the Influence of applied electric field "F" and is given by

$$J_n = n e V_{dn} \quad —① \quad J_n$$

Where, V_{dn} → drift Speed of e^-s. and is expressed as

$$V_{dn} = \mu_n F \quad —②$$

If σ_n → conductivity of Semi Conductor materials (conductivity) due to free e^-s. Then current density J_n due to drift e^-s. is also given by $J_n = \sigma_n F \longrightarrow ③$

$\boxed{J = \sigma_n E}$

F

from eqn ① & ②

We have

$$\sigma_n F = n e V_{dn} .$$

from ③.

$$\sigma_n F = n e \mu_n F$$

$$n e \mu_n = \sigma_n \longrightarrow ④$$

IIly In VB, if P density of holes.

If $P \rightarrow$ density of holes Then σ_P, the conductivity due to holes is

$$\sigma_P = P_e \mu_P \longrightarrow ⑤$$

Where $\mu_P \rightarrow$ Mobility of holes.

Total conductivity is

$$\sigma = \sigma_n + \sigma_P .$$

$$= n e \mu_n + \mu_P P_e .$$

$$\sigma = e (P \mu_P + n \mu_n)$$

for intrinsic semi conductor density of holes in VB equals the density of electrons in CB.

$$n = p = n_i \quad \sigma = e n_i (\mu_n + \mu_P).$$

where n_i no. of e's or holes per unit Volume in The materials and is given by $n_i = \int Z.(E) \underline{F(E)} . dE$

Impurity states

In impurity semiconductor, impurity atoms however reduce the energy gap (Eg).

Donors $\xrightarrow{\text{represent}}$ isolated energy levels located so close to unfilled band That very little energy is required to lift an e⁻ from the donor level into the unfilled band that very little energy is required to lift an e⁻ from the donor level into the unfilled band (conduction band) where it is available for the conduction of electricity.

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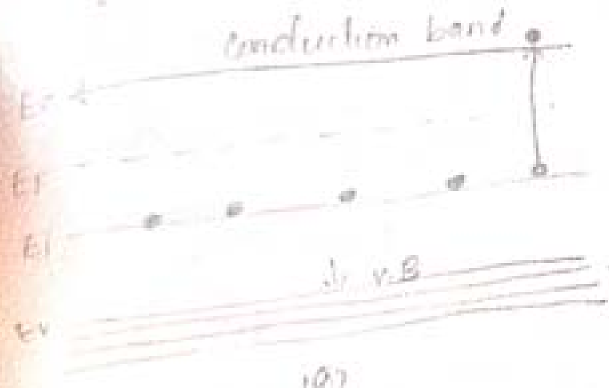
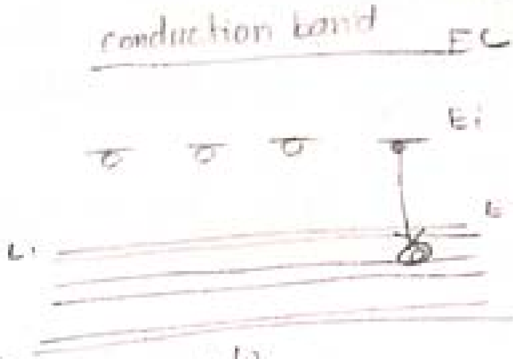


(a)

donor level



(b)

acceptor level

Then in that case of density of e̅s in C.B will be given by eqn,

$$n_c = 2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2} \exp\left[E_F - E_C)/k_B T\right] \quad \text{①}$$

Assume that

E_F lies more than a few k_BT above the donor level Then the density of empty donors is given by. Nd

$$N_d\left[1 - F(E_i)\right] \approx N_d \exp\left[(E_I - E_F)/k_B T\right] \quad \text{②}$$

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from eqn ① & ② We get.

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$$\frac{\exp^{(E_F-E_C)/K_BT}}{\exp^{(E_i-E_F)/K_BT}} = \frac{N_d}{2\left[\frac{2\pi m_e^* K_BT}{h^2}\right]^{3/2}}.$$

$$\exp^{(E_F-E_c-E_i+E_F)/K_BT} = \frac{N_d}{2\left[\frac{2\pi m_e^* K_BT}{h^2}\right]^{3/2}}.$$

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$$2E_F-(E_c+E_i)/K_BT = \log\left[\frac{N_d}{2\left[\frac{2\pi m_e^* K_BT}{h^2}\right]^{3/2}}\right]$$

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$T = 0K$

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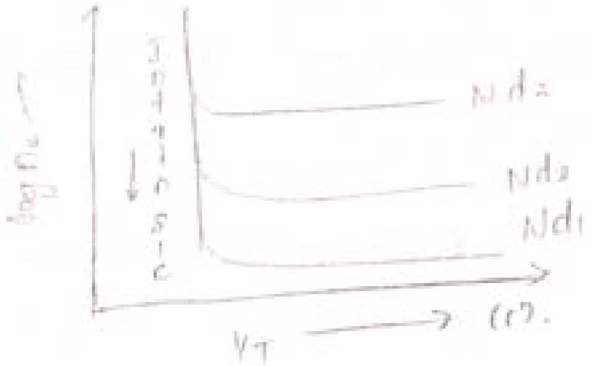

The above figure shows that the variation of conduction e⁻ density with Temperature.

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from eqn ① & ③

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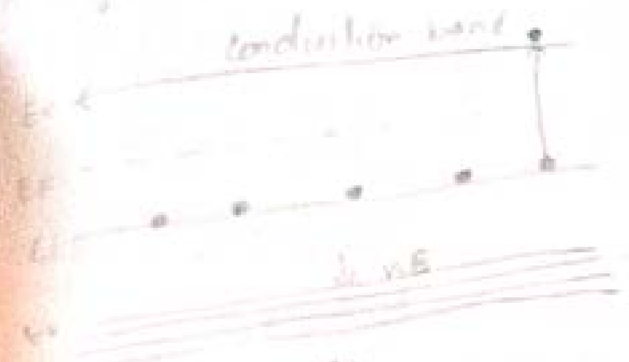

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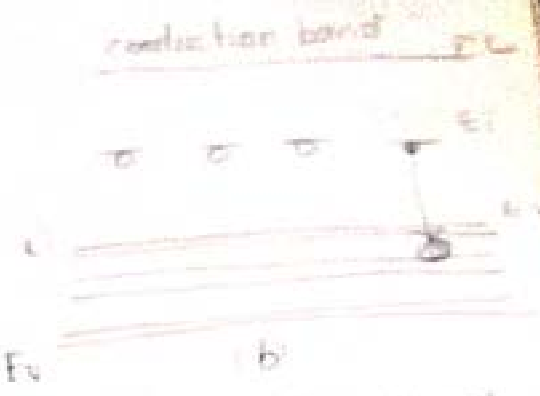

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$$N_d \exp\left[E_I - E_F)/K_B T.\right.$$

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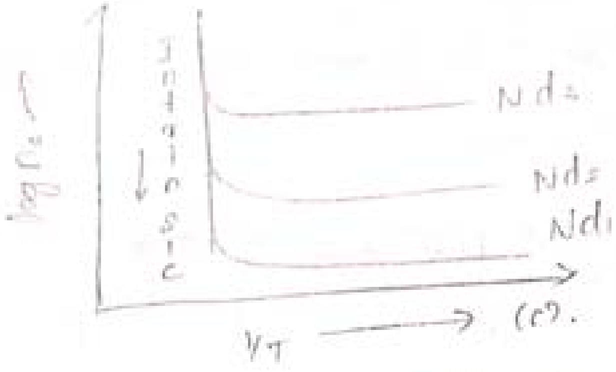

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$$= \exp\left[\frac{E_i - E_c}{2K_BT}\right] \cdot \frac{(N_d)^{1/2}}{\left[2\left(\frac{2\pi m_e^* K_BT}{h^2}\right)^{3/2}\right]^{1/2}}$$

$$n_c = 2\left[\frac{2\pi m_e^* K_BT}{h^2}\right]^{3/2} \frac{(N_d)^{1/2}}{\left[2\left(\frac{2\pi m_e^* K_BT}{h^2}\right)^{3/2}\right]^{1/2}} \cdot \exp\left[E_i - E_c / 2K_BT\right].$$

$$(2N_d)^{1/2}\left[\frac{2\pi m_e^* K_B T}{h^2}\right]\exp\left[E_i - E_c/2k_BT\right]$$

Where $\Delta E = (E_c - E_i) \to$ ionisation energy of donars.

$$n_c = (2N_d)^{1/2}\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/4}\exp^{-\Delta E/2k_BT} \quad ①$$

Note That :-

Density of e^- in CB is $\propto \sqrt{con}$.
Proportional to square root of con.

As the temperature increases, fermi level falls below the donor level and it approaches the centre of forbidde gap which makes the substance an intrinsic semiconductor.

Conductivity of intrinsic semiconductor is smaller than n or p type semiconductors.

In fig C

A graph is plotted in $\log n_c$ and $1/T$ Which gives a straight lines with a slope $-\Delta E/2K_B$.

If the T becomes sufficiently high so as to excite $\bar{e}$'s from VB to reach directly to CB, then this slope changes to $-E_g/2K_B$.

Electrical conductivity

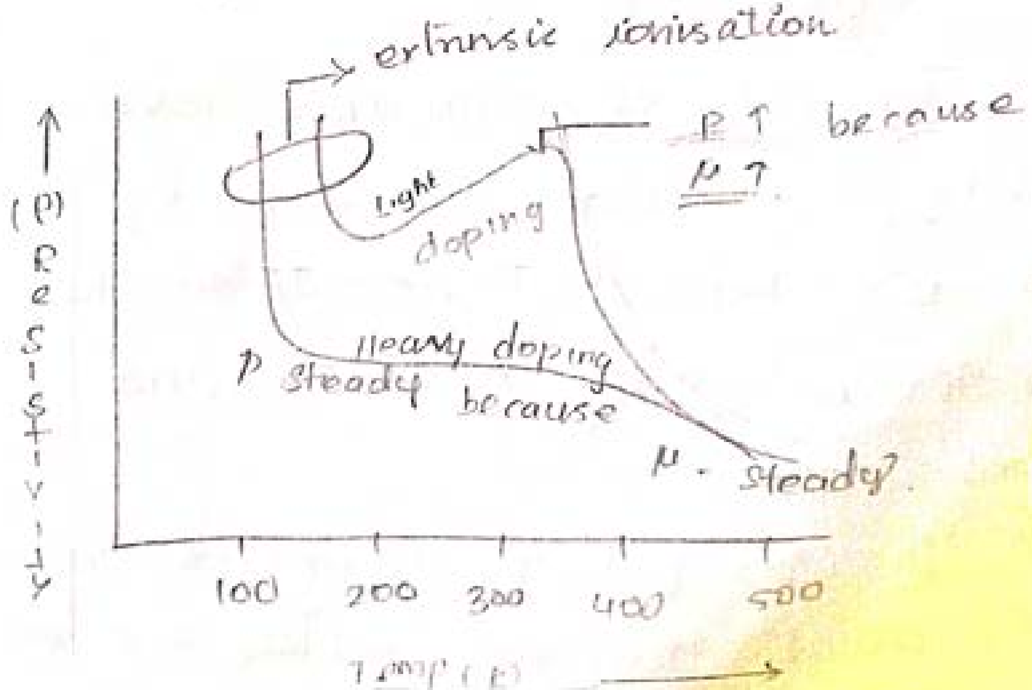
For pure n-type Semiconductor in which acceptors are not present. The electrical conductivity can be written as.

$$\underset{\text{conductivity}}{\sigma} = e n_c \mu_n \quad (\mu_h = 0).$$

Where

$\underset{\text{density}}{\underline{n_c}}$ is given by eqn ④

By using that study the variation of conductivity with temp.



The above figure represents temper variation of resistivity for heavily & light doped semi conductors.

p- type Semiconductor Refer fig (b)

In this case acceptor levels lie ab the valence band.

The density of holes in the Valence ba can be obtained exactly in the same wa as in the case of nc

$$n_h = (2N_d)^{1/2}\left[\frac{2\pi m_h^* k_B T}{h^2}\right]^{3/4} \exp\left[-\Delta E/2k_B T\right] \quad ⑤$$

where

$$\Delta E = E_i - E_v.$$

In this case fermi level lies half way b the acceptor levels and top of valence band at T=0k.

When density of available level's Z_i with energy E_i are occupied by density of donor e^-s, N_d at T=0k as in the case of Ge & Si containing trivalent and Pentavalent impurities

But if $Z_i >> N_d$ as happens in the case of alkali halides containing excess metal then only a fraction of available evels will be Occupied

In this case, at any Temp T. no of filled impurity levels is equal to

$$N_d = \frac{z_i}{\exp[(E_i - E_F)/K_BT] + 1}$$

When $(E_i - E_F) > $ a few K_BT

$$N_d = z_i \exp[(E_F - E_i)/K_BT] \quad \text{—— ⑥}$$

from eqn ① We have.

$$n_c = 2\left(\frac{2\pi m_e^* K_BT}{h^2}\right)^{3/2} \exp[E_F - E_c | K_BT]$$

$$= 2\left(\frac{2\pi m_e^* K_BT}{h^2}\right)^{3/2} \exp\left[(E_F - E_i)/K_BT\right]$$

$$\cdot \exp[E_i - E_c) / K_BT]$$

$$= 2\left(\frac{2\pi m_e^* K_BT}{h^2}\right)^{3/2} \left(\frac{N_d}{z_i}\right) \exp(E_i - E_c/K_BT)$$

on Using eqn ⑥

$$\Delta E = (E_c - E_i)$$

We find that density of e^-s in CB is given by,

$$n_c = 2\left(\frac{N_d}{z_i}\right)\left(\frac{2\pi m_e^* K_BT}{h^2}\right)^{3/2} \exp(-\Delta E/K_BT)$$

in which exponential term consist of ΔE instead of $\Delta E/2$ as compared to eqn ②

$$n_c \propto N_d$$

Temperature dependence ,

for intrinsic semi conductors

$$n = p = n_i$$

$$E_F = \frac{1}{2}(E_c + E_v)$$

$$n_i^2 = N_c N_v e^{-(E_c - E_v)/kT}$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/kT}$$

Where,

$E_c - E_v = E_g \rightarrow$ Band gap.

general $$n_i = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}(m_e^* m_h^*)^{3/4} e^{-E_g/2kT}$$

Experiment results shows that the band gap for most of the semiconductors decreases linearly with term so that we may write,

$$E_g = E_{g_0} - \alpha T$$

Where,

$E_{g_0} \rightarrow$ Bandgap at 0k and

$\alpha \rightarrow$ constant.

characteristic of the Specific semiconductor eqn ① can be rewritten in the following form.

$$n_i = 2\left(\frac{kT}{2\pi \hbar^2}\right)^{3/2} (m_e^* m_n^*)^{3/4} e^{-(E_{g0}-\alpha T)/2k}$$

$$= AT^{3/2} e^{-E_{g0}/2kT} \text{———} ②$$

Where,

$$A = 2\left(\frac{k}{2\pi \hbar^2}\right)^{3/2} (m_e^* m_n^*)^{3/4} e^{\alpha T/2kT}$$

eqn ① & ② pts to the fact that the carrier concentration in intrinsic semiconductor is strongly tem.-depend

$$e^{-\frac{E_g}{kT}}$$